THE
CORONATION DATE OF
Queen Elizabeth I
AND MORE MONAS Mathematics
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## THE

# CORONATION DATE OF 

Queen Elizabeth I
AND MORE
MONAS
Mathematics

BY<br>JIM EGAN<br>Cosmopolite press<br>Newport, Rhode Island


"Citizen of the World"
(COSMOPOLITE, IS A WORD COINED by John Dee, from the Greek WORDS COSMOS MEANING "WORLD" AND POLITÊS MEANING "CITIZEN")

## Dedication

To my main cheerleaders:
Edward Egan and Dot Ford, Robert and Louise Egan, Barbara and Ned Landon Gail and Don Niermeyer, Allen Green, Pam and Gerry Fracareta Mary Dutt, Jane Christopher, Amita and Viren Mehta, Michael Stineman and Purna Rodman,

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My production assistants:
Rus Benson, Elaine Madden, Jeanine Costa-Gringevich, and Liz Stevens,
And those who helped with some of my translations:
Scott Barker (Latin) and Peter Lech, René Nünlist (Greek)

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## Why John Dee SELECTED JANUARY 15 TO BE THE DATE FOR THE CORONATION of Queen Elizabeth I

When Queen Mary I died in November of 1558, Robert Dudley asked the wise Dee to "to name an auspicious day" for Elizabeth I to be crowned.
(Charlotte Fell Smith, John Dee, p. 18)
Nobody has ever been able to explain why he chose January 15, 1559 as the Coronation Day. But after understanding the Monas, one can easily see the reason. This this particular date involves Metamorphosis numbers 12, 24, and 360. Can you figure out how?

First, let's turn the clock back 3000 years. The ancient Babylonians counted the days of the year and came up with 365 . (They were pretty close; it's actually 365.2425 ).

They knew $5 \times 73$ equaled 365 , but 73 days was too large a time period to deal with. Instead, they rounded 365 off to 360 , a number that could be divided up in many ways.


They chose to have 12 months of 30 days each and tagged the five extra days on at the end of the year. (This use of 360 could be what led them to use the Base 60 for their arithmetic.) And as noted previously, several of the authors of the Bible rounded the year off to 360 days as well.

Here is the 365.24 -day-year broken up into 12 parts of 30 days, plus a 5.24 snippet at the end. As fewer and fewer people are speaking Babylonian these days, I've used the more familiar Roman names (despite the fact that they're not all 30 days long).


Next, let's divide the 30 day months in half, making 24 periods of 15 days each. Dee's selection for the for the Queen's coronation date is exactly one twenty-fourth of a year!


This way of looking at January 15 involves the 3 Metamorphosis numbers 12, 24, and 360. But wait, there's more!

Let's enlarge the January 1 to January 15 section of this calendar.

In each day there are 24 hours, so over a 15 -day period, that makes 360 hours!

Another correspondence with 360 .
No wonder the liked this date!


This next circle shows that in the first 15 days there are 360 hours. (Incidentally, there are $360 \times 24$ hours, or 8640 hours, in a year that is 360 days long.)

Let's visualize this another way, using what Dee calls "Circular Arithmetic," by dividing a circle into 24 parts.

Each part has 15 degrees (or 15 days) in it. Twentyfour parts of 15 degrees each makes a 360-degree circle.

When Dee was asked to select the date for the Queen's coronation, I don't think it took had to think about it for too long. He had fully developed his mathematical cosmology by then, as he had incorporated it in his 1558 Propaedeumata Aphoristica (which also cryptically references his Greek-titled works on Metamorphosis and Consummata, as I explained earier).

But there is more evidence suggesting that the Dee had embraced January 15 even before 1556.

## The date of Dee's Royal Library proposal

When Mary became Queen in 1553, Dee was tossed in jail. To save his skin he became a Catholic priest under Bishop Bonner. Over the next few years, Dee got more comfortable with Mary's regime and became bold enough to even present a proposal to her.

With the dissolution of the monasteries under her father Henry VIII, many of the books and manuscripts from monastic libraries had been dispersed and lost. Dee penned a "Supplication for the recovery and the preservation of ancient Writers and Monuments," recommending that Queen Mary I establish a Royal Library. Here is an excerpt:

> "But albeit that in those days many a precious jewel and ancient monument did [utterly perish (as] at Canterbury did that wonderful work of the sage and eloquent Cicero de Republica, and in many other places and like)."
(Dee, Supplication, in Roberts and Watson, p. 194)
Dee volunteered to assemble the library, even to scour the Continent in search of rare texts. And the whole project was to be carried out "without any penny charge unto your [Majestie]."

Dee's proposal fell on ears is as deaf as the one as the ones that she and bloody Bonner were busy cutting off.

But the important clue as this:
Dee dated his "Supplication"
January 15, 1556
Dee undoubtedly felt the proposal would be accepted and this would be the "creation date" of an institution that would grow over the centuries to become an important repository of wisdom (as the British Library later became in 1753)

This date, January 15,1556 involves the cosmologically important number 24 in a different way. If the digits of the month (1), the day $(1+5)$ and the year $(1+5+5+6)$ are added, they sum to 24 . Having the numerical power of the cosmos on his side probably emboldened Dee to approach the Queen on this day.

To Dee the idea of preseving the great written works of the ages in a national library was a no-brainer. But on this issue, unfortunately, this seems to be what Mary had.

## A quick review of Dee's use of 12, 24, and 360

Dee hints about the importance of 12,24 , and 360 in many places.

The Vessels of the Holy Art diagram in Theorem 22 indicates he applied these numbers to the Monas symbol, as the two arms of the Cross are labeled M, the twelfth Latin letter.


We can see it on the Title page where the theater is $24 \times 24$ grid-squares, which puts the center point 12 grid squares in from each of the 4 sides.

In the "restored" Title page, that is also the center point of the Sun Circle, which is a 360 degree circle.

12252240
1225224 O

We can see 12,24 , and 360 in the "Thus the World Was Created" chart, when it has been "ballooned to 360 ," making the two circle design. Also, 12 and 24 are displayed prominently within the chart.


A graphic way to see 12,24 , and 360 in the Exemplar number is to envision the final zero as a circle.

But a more mathematical way is to add the first 3 Metamorphosis numbers. When that sum of 108 is added to the 252 , then 360 has been obtained arithmetically.



Exerpt from the Letter to Maximillian
(p. 8, emphasis mine)

The numbers 12,24 , and 360 are woven into the very fabric of Dee's book.He even informs us that the "Quality" of his gift "is defined by its own limits."

Dee tells us that his 24thTheorem that it is "like a Circle Completing Itself."


THEOR. 24
"In the Beginning of this Little Book, we started with a Point, a Straight Line, and a Circle.

## Now, at the End,

like a Circle Completing Itself, we have a POINT, LINE, and our ELEMENTS Flowing Out of our MONAD, which is Analogous to the Equinoctial when a Circuit is completed in 24 Hours."

Exerpt from Theorem 24
(p. 27 verso, emphasis mine):

This "circlegram" shows 24 sections in a 360 degree circle shows the book "completing itself."

Theorem 12 is "half-way around." (And note that Theorem 1 is much like January 15, the Queen's Coronation Date, one twenty-fourth of a circle).

Dee is quick to add that this way of seeing the book is like the "Equinoctial when a Circuit is completed in 24 Hours."


In Theorem 11, Dee emphasizes that in the "Mystical Sign of Aries" there are 12 hours of lightness and 12 hours of darkness.

In several other Theorems he indicates that the two horns of Aries can be morphed into a 360 -degree circle.



So the Equinox might be visually summarized by a circle that is half dark and half light.

At the latitude of the Tower, the dark and light line is actually at an angle, but it still incorporates 12, 24, and 360.


And of course 12, 24, and 360 can easily be seen in the design for the John Dee Tower.

Half of the 48-foot tower is 24 feet. Half of that 24 feet is the 12- foot height of the "pillars plus the pillar entablature."

A bird's eye view of the Tower shows that it has a 12 foot radius, a 24 foot diameter, and is round, like a 360 degree circle.


Standing in Touro Park, looking at the timeworn remnants of the Tower, it doesn't seem likely that it expresses the Coronation Date of Queen Elizabeth I. But both of these things are constructions come from Dee's mind, and he certainly was consistent about applying his mathematical cosmology in diverse fields.

## THE 252 PRETZEL EXTENDED <br> INTO A SPIDER'S WEB



Some of the harmony of the 12 and 21 Pretzel can be seen in it's neighbor, the 13 and 31 Pretzel.
The squares of 13 and 31 are transpalindromes (169 and 961).


But after 13 and 31, the pretzel crumbles.
The squares of " 14 and 41 " are not transpalindromic mates (196 and 1681).


The squares of 15 and 51 are not transpalindromic mates (225 and 2601).

You can see by this trend that no more "perfect pretzels" will be found in the realm of numbers.


This might make it seem as though " 12 and 21 " and " 13 and 31 " both win gold medals.

But, a deeper look into their pretzels will show that " 12 and 21 " gets the gold medal, and " 13 and 31 " is relegated to a silver medal.

## "Extended" Pretzels are better viewed as Spider webs

As a "pretzel" is sort of a
"double-lobed complete circuit," it's hard to grow "branches" off of it.

For a deeper look into the pretzel, let's flatten it out into a "spider web."


Now, we can add a depiction of the ratios "between" the members of the pretzel. The ratio between 144 and 441 is $16: 49$, which is (4x4): (7x7).

Look what happens when we add 252 to the squares of 252 's components, 12 and 21.
$252+144$ makes 396 . $252+441$ makes 693 .

Not only are 396 and 693 transpalindromes, they are also in the $4: 7$ ratio!

$$
(396 / 693=4 / 7)
$$

(Clue: You might recognize 396 and 693 as members of the 99 Wave (as $99 \times 4=396$ and $99 \times 7=693$ ).

Just as significant is the fact that they sum to 1089.
And we just found them in the investigaton of the Exemplar Number)

Next, let's extend this growing "spider web" in another direction.

If we add 108 to 252 we get 360 .
If we subtract 189 from 252 the result is 63 . And 63 and 360 are transpalindromic mates!

This is one wondrous web of retrocity!


Here is a similar web done with 13 and 31.

Note that at the bottom, 572 and 1364 are not transpalindromes.

And at the top 637 and -155 are certainly not transpalindromes.
(This is why 13 and 31 only get a silver medal compared to 12 and 21 's gold medal.)


Here, in the web of 14 and 41, all transpalindromicity among these sums has vanished.

Likewise, it has vanished in the web for 15 and 51.


I suspected that perhaps the transpalindromicity would recur at " 24 and 42 ," but alas, it doesn't.
(However, one interesting result is in the upper right-hand corner of this chart where $1080-756=252$.
Dee's Magistral Number keeps popping up.)

The only other numbers that form pretzels besides (12 and 21) and (13 and 31) are (33 and 99). Though (33 and 99) are not transpalindromes (they are each palindromes), their squares are key numbers in the 1089 Wave.

We'll investigate this shortly, but as a hint, remember that $12+21=33$.


Likewise in the web of 16 and 61
..... And beyond.....


To get a clearer picture of what's going on in this mathe-magical " 12 and 21 " spider web, let's look at the results in terms of $\mathbf{1 2}$ and 21.

Note that along the bottom, " $12 \times 33$ " is a transpalindrome of " $21 \times 33$."
And at the top, " $12 \times 30$ " and " $21 \times 3$ " are also transpalindromes.


Looking at the "3:4 ratio x 3" boxes (which are the $9: 12$ boxes), we can find these related numbers, 3,30 , and 33 .


Now, let's look at that same " 12 and 21 " spider web in terms of the number 9 .
Along the bottom row, the transpalindromes 396 and 693 and along the top row, the transpalindromes 360 and 63 are made from the " 9 " times numbers which involve ony the numbers " 4 and 7" (that is, $7,40,44$, and 77).


These numbers which keep popping up, (9, 63, 36, 99, 396 and 693), are saturated with 3 's, 6's and 9's.

There's something unusualabout this small coterie of numbers which might be metaphorized this way::

## If 3,6 , and 9 are oil ..... 4 and 7 are water

In the decimal part of these "fractions of 7" there is not a 3,6 , or 9 to be found
$1 / 7=.142857142857142857 \ldots$
$2 / 7=.285714285714285714 \ldots$
$3 / 7=.428571428571428571 \ldots$
$4 / 7=.571428571428571428 \ldots$
$5 / 7=.714285714285714285 \ldots$
$6 / 7=.857142857142857142 \ldots$
$7 / 7=1$
$8 / 7=1.142857142857142857 \ldots$
$9 / 7=1.285714285714285714 \ldots$
$10 / 7=1.428571428571428571 \ldots$
$11 / 7=1.571428571428571428 \ldots$
$12 / 7=1.714285714285714285 \ldots$
$13 / 7=1.857142857142857142 \ldots$
$14 / 7=2$
$15 / 7=2.142857142857142857 . .$.
$16 / 7=2.285714285714285714 \ldots$
$17 / 7=2.428571428571428571 \ldots$
$18 / 7=2.571428571428571428 \ldots$
$19 / 7=2.714285714285714285 \ldots$
$20 / 7=2.857142857142857142 \ldots$
$21 / 7=3$
$22 / 7=3.142857142857142857 \ldots$
$23 / 7=3.285714285714285714 \ldots$
$24 / 7=3.428571428571428571 \ldots$
$25 / 7=3.571428571428571428 . .$.
$26 / 7=3.714285714285714285 \ldots$
$27 / 7=3.857142857142857142 \ldots$
$28 / 7=4$
$29 / 7=4.142857142857142857 \ldots$
$\left.\begin{array}{ccc}30 / 7 & =4.285714285714285714 \ldots \\ \ldots & \ldots & \ldots\end{array}\right)$

To demonstrate what I mean, let's look at the "fractions of 7" when they are seen as decimals.

All the results in this chart are " 6 -digit repeating decimals" that involve the numbers

$$
1,4,2,8,5, \text { and } 7
$$

But in the never-ending repeating decimal part, 3's, 6's, and 9's never appear!

Likewise, the decimal parts of the "fractions of 4," are comprised of 2 's, 5 's, and 7's, but there are no 3's, 6's, or 9's.

The discovery of decimal fractions is usually credited to the Belgian mathematician
Simon Stephen and 1585. Dee does not mention decimals in his Preface to Euclid, nor does he use them in his work. Whether Dee used decimals is not really that significant here.

This repeating pattern, (which lacks 3's, 6's, and 9's) can easily be seen
by dividing 7 into any "high multiple of 10 ," like a billion or a trillion

$$
7 \longdiv { 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 \ldots }
$$

[We know that Dee is not afraid to think in terms of high numbers like this. He uses 1 million $(1,000,000)$ and 1 billion $(1,000,000,000)$ and his Arbor Raritatis illustration.

We know that he loved the number 7 ("the remarkable septenary").
In the same chart, he shows a man's life is divided into 7-year periods.]

It may seems like (7) and (3's, 6's, and 9 's) are opposites like oil and water, but in another way they create wonders when they work together, for example:

7 mixes with 36 to make 252,
7 mixes with 360 to make 2520 , and 7 mixes with 3690 to make 27720 .


To extend the metaphor,when you take a bottle of salad dressing from the refrigerator, you'll see all the oil (3's, 6's, and 9's) floating on top of the water (4's and 7's).

But after a few vigorous shakes, you've got some tasty salad dressing.
As another example of oil and water mixing, add the decimals for $3 / 7$ and $4 / 7$.

Suddenly, its all about 9 .
It seems like this never-ending series of 9's will never reach 1. But, as $3 / 7+4 / 7=1$, we all know the real end of the story.

$$
\begin{aligned}
& 3 / 7=.428571428571428571 \ldots \\
& \frac{4 / 7}{7 / 7}=\frac{.571428571428571428 \ldots}{.999999999999999999 \ldots}
\end{aligned}
$$

(This characteristic is not exclusive to $3 / 7+4 / 7$, as $1 / 7+6 / 7$, or $2 / 7+5 / 7$ results in a string of 9 's as well.)

As a final example, when the water of $4: 7$ is mixed with 3,6 , and 9 (and also 12) the result is Dee's " 4 steps" and their transpalindromic mates!


Dee was aware of this dynamic between ( 3,6 , and 9 ) and (4 and 7).
He expresses it in the proportioning of the Monas symbol, which he suggests can be made from either 10 points or 9 parts.


To summarize, here's a web of all the proportional interrelationships among the members of the "extended spider web pretzel."

It's comprehensive, but dizzying to look at.


To graphisally simplify, let's untwist the pretzel in an even different, more linear way.
To start with, here I have graphically expressed the 3:4 "part to part" ratio as a
"trio of gracefully leaping arches" (a family of geometric gazelles).

The 3:4 "part to part" ratios
seen as "leaping arches"

The 3:4 ratio times 1

The 3:4 ratio times 2

The 3:4 ratio times 3


Here's what the " 12 and 21 " pretzel looks like when seen as as "leaping arches." I've inverted one set of arches to emphasize how 252 is involved in both sets.


Finally, here are the same leaping arches involved with those imporant numbers 396 and 693 (which are in the 4:7 proportion, are transpalindromic mates, and sum to 1089).


The table-charts, pretzels, spider webs and leaping arches presented in this chapter are simply different ways of looking at the same complex, yet thrilling, mathematical interrelationships (like watching a touchdown pass from various camera angles).

It's the numbers themselves that are so wondrously interwoven.

## MORE MAGICAL MATHEMATICICS OF THE MONAS

The Monas symbol ties all of Dee's mathematical cosmology together. This innocent-looking, one-eyed homunculus made from geometric shapes actually integrates the varied mathematical wonders of 252 , Consummata (including the " $+4,-4$, octave, the 9 wave, the 11 Wave, the 99 Wave, the 1089 Wave, ...), the Metamorphosis numbers and even the Exemplar Number, 12252240. That's one tall order. Let's see how.


Standing at attention with its perfect symmetry, the Monas Symbol seems so prim and proper.

But looking at it in a "mathematical way," I see the symbol doing a wild gyrating dance, flipping between the "oil of 3,6 , and 9 " and the "water of $1,4,7,10$."


My intent here is not to distort or show dirsespect for Dee's harmonious proportions, but to encourage "flexible" thinking about the mathematical stories it can tell us.

First, let me review the references Dee makes in the Monas Hieroglyphica that justify my vision of the Monas symbol doing this wild tarantella.

Just outside the lower left-hand corner of the "Thus the World was Created" chart Dee writes,
"Ancient enigma of the Symmetry of the Decad explained."

Also, in the bottom half of the chart, he highlights $\mathbf{1 , 4}$, and 7 by enlarging them and putting them
 in their own separate column.

In Theorem 23, Dee gives a detailed construction of the Monas symbol from which it can be concluded that it has $\mathbf{1 0}$ points on its vertical spine.


In the same Theorem, he also remarks that the spine is made from "nine equal parts" thus giving the "Mechanicum another way" to "go about performing this work."

Dee drops a hint about these " 9 parts" in Theorem 21 while
 describing 3 orientations of the Aries symbol. He adds that they might also be seen as $\mathbf{6}$ things (that is, 6 half-circles). Then, he says 3 and 6 "Summed together make three times three." (This is his subtle way of saying nine without actually saying it.)
"Three groups of 3 " is the most obvious way to divide nine parts. But there's another important way to divide nine parts: " 2 groups of 4 " plus "one." In other words, Bucky's " $+4,-4$, octave; null 9 ," the natural rhythm of Base Ten.


These two ways to group the "9 parts" are but a "sneak preview" of where this chapter is headed.

## But first, let's concentrate the "10 points" of the Monas symbol.

I've explained previously how the " 1 , $4,7,10$ " of the Symmetry of the Decad might be seen as the " $10,40,70,100$ " of what I call the "Symmetry of the Centad," or the "100, 400, 700, 1000" of the "Symmetry of the Milliad," or beyond. This is a fairly formal regimentation of how the symbol relates to the Base Ten.


Now, recall the way the 9 Wave appeared naturally in the "diamond-shaped chart of 1and 2-digit numbers." (The chart concludes at 99 and does not include that 3-digit number, 100) The chart's horizontal axis includes 10 numbers ( $9,18,27,36,45,54,63,72,81$, and 90 ).

Here, I've stacked those numbers up next to the 10 points of the Monas Symbol and highlighted the octave of 9 Wave transpalindromes, (18 and 81), (27 and 72), (36 and 63), and (45 and 54).

I've super-highlighted the numbers 36 and 63 . Notice how they correspond with the 4 and the 7 respectively. This is because 36 is ( 4 x 9 ) and 63 is ( $7 \times 9$ ). Thus, 36:63 is equivalent to 4:7.


Even though we're dealing with the " 10 points of the Monas Symbol (not the " 9 parts" yet) these 3's and 6's are starting to become involved. (Also, 36 and 63 sum to 99 , which is the 9 times 11 (the "transpalindromizer" of Base Ten times the "palindromizer" of Base Ten).

Next, let's push into the "realm of 3-digits" and associate the 99 Wave $(99,198,297,396,495$, $594,693,792,891,990$ ) with the " 10 points" of the Monas Symbol.

Again, I've highlighted the octave of 99 Wave transpalindromes (198 and 891), (297 and 792), (396 and 693), (495 and 594).

And, I've super-highlighted 396 and 693. Here's more of that oil of " 3,6 , and 9 " relating with the water of " $1,4,7$, and 10 ."

Recall that 396:693 is equivalent to 4:7, as 396 is ( $4 \times 99$ ), and 693 is ( $7 \times 99$ ). They sum together to 1089 , which is ( $11 \times 99$ ).


You will recognize these "oily numbers" 36, 63, 396, and 693 from the "extended spider-web pretzel" of "12 and 21," the heart of which is Dee's Magistral number 252.

We arrived at 396 by adding 252 to 144 . ( $12 \times 21$ ) plus ( $12 \times 12$ ) equals $(12 \times 33)$
We arrived at 693 by adding 252 to 441 . ( $12 \times 21$ ) plus ( $21 \times 21$ ) equals $(33 \times 21)$.
On the top of the spider-web,
We added 108 to 252 and got 360 . ( $12 \times 9$ ) plus ( $12 \times 21$ ) equals ( $12 \times 30$ )
We subtracted 189 from 252 and got 63 . ( $12 \times 21$ ) minus ( $9 \times 21$ ) equals ( $3 \times 21$ )


Admittedly 360 is not the same thing as 36 . This appears to be a "flaw" in the system. However, using retrocity eyeglasses, one can see that those great numbers $\mathbf{3 6}$ and $\mathbf{3 6 0}$ are quite related in a way other than "a factor of $10 . "$ They both have the same transpalindromic mate, $\mathbf{6 3}$.

360 A 63
36 A 63

If you'll allow me to detach that zero from 360 to make 36 , then we have the full set the 4 very important numbers derived from 252 , (that is, $36,63,396$, and 693) all aligning with the " 4 and 7 points" of the Monas Symbol. In addition, they are all key members of either the " 9 Wave" or the "99 Wave." That's pretty amazing synchronicity!

Remember, 252 may not be a Metamorphosis number, but it's the product of 12, (the first Metamorphosis number), and its reflective mate, 21. $12 \times 21=252$ Also, remember that 252 is the reflective mate of 2520, a key Metamorphosis number, being the lowest number divisible by all the single digits.

2550 -252

To summarize, the Monas Symbol can be seen as a "bridge" between Metamorphosis and Consummata, helping to us to see their wondrous "synchronicity."

After all this harmony found in the "1-digit, 2-digit and 3-digit ranges" of number, suddenly when we step into the "4-digit, 5-digit and 6-digit ranges" (and beyond), something jarring happens! Suddenly, a nave is born.


In the 4-digit range, the octave of 1089 Wave transpalindromes, (1089 and 9081), (2178 and 8712), (3267 and 7623), (4356 and 6534) has the number 5445 ( $1089 \times 5$ ) as a nave.

A similar thing happens in the 5-digit range of number, where $54450(10890 \times 5)$ becomes a nave.

Similar naves as are found in the 6 -digit, 7 -digit, and 8 -digit ranges, and beyond...
(Perhaps you can explore why this phenomenon happens. It's not exclusive to Base Ten. Similar "switches" happen in all of the bases.)

In these larger ranges of number, the "octave of transpalindromes" plus their "nave" makes for 9 numbers (not 10 anymore). Notice that the numbers 1089, 2178, 3267, 4356, 5445, $6534 \ldots$ all fall in the spaces, between the dashed lines.

Another unusual feature of this arrangement has to do with the midpoint of the members 9 Wave. Halfway between 45 and 54 is $\mathbf{4 9 . 5}$. Likewise, halfway between 36 and 63 is $\mathbf{4 9 . 5}$. This is also the average of " 27 and 72 ", and also of " 18 and $81, "$ as $\mathbf{4 9 . 5}$ is half of 99 .


On the Monas Symbol, the 49.5 midpoint (the 499.5 midpoint in the 3 -digit range), the 5445 nave, the 54450 nave, the 544500 nave, ... all intersect the Monas Symbol's spine at an obscure point inside the Sun Circle (which I have marked with a star). Dee makes no reference to this point in his geometric construction of the Monas.

## What's going on here and how might it be resolved?

A resolution can be found by shifting the whole chart (downwards by a "half of a part"), so that the 9 Wave's transpalindromes and the 99 Wave's transpalindromes align with the " 9 parts" rather than the " 10 points."


Look what happens now!
The "star" now falls at "point 5," where the Cross of the Elements intersects the Sun Circle. This seems to be a more appropriate position for the naves, 49.5, 499.5, 5445, 54450,...

On the extreme left of the chart, I've included the single digits. See how nicely Bucky's " $+4,-4$, octave; null 9 " rhythm fits the chart.

In short, this is that (4 and 4 and 1) pattern which we had a "sneak preview" of earlier.


At the risk of sidetracking you into confusion, let me briefly remind you how we saw "octave, nine" in 2 related concepts.

You'll recall that the $8: 9$ is "epogdous," the ratio of the 2 middle members of Nicomachus' and Boethius' "most perfect harmony," ( $6,8,9,12$ ).

You'll also recall that 8 tetrahedra (4 pairs) sharing a common, "locus of vanishment"(the "null 9" centerpoint) form a cuboctahedron.

The beauty of this " $8+1$ " arrangement is that now the two circles of the Monas Hieroglyphica, the Sun and the "Moon that yearns to transform itself into the Sun", now fit exquisitely in the lower 8 parts. One is in the identical position as the Sun Circle. The diameter of the other circle is the "vertical axis" of the Cross.

Each circle has the height of " 4 parts," nicely expressing the " $+4,-4$, octave." But on top of the that, the two tangent circles actually form a figure $\mathbf{8 !}$ (In the Monas, the figure 8 can be made graphically by superimposing the in the 2 "flowing ribbons" of the Title Page. It can also be found by folding the "final-page emblem" back-onto-itself. Dee also seemed to know that the "3 main motions" of the earth with respect to the Sun generate a figure 8 -shaped analemma.)


Remember that the " 2 circles" are the overriding design feature of the "ballooned 360 " Thus the World was Created" chart.

And even more importantly, " 2 circles" are the main design plan of the John Dee Tower!
But even more importantly, when we assemble an "octave of Metamorphosis numbers" the eighth number is 6126120 , which is evenly divisible by $1,2,3,4,5,6,7,8,9,10,11,12,13$, $14,15,, 17$, and 18 . To include 16 in the mix, it "needs 2," making it 12252240, the Exemplar number. This is one important way in which the John Dee Tower represents 12252240.


So what's the right way to correspond Consummata with the Monas symbol?
Is it with its " 10 points" or with its " 9 parts" ?
The answer seems to be: BOTH.
Each method has its own harmonious correspondences.
Is it better to see the " 9 parts" as " 3 groups of 3 ," or as " 8 plus 1 "?
Again the answer seems to be: BOTH.
These questions bring to mind
something which Dee wrote about
right on the Title Page of the Monas Hieroglyphica
(albeit cryptically).
How does the 11 Wave relate to these various proportionings of the Monas Symbol?


Finally, let's see how the "11 Wave" fits into this scenario. Here is one way that the palindromic " 11 Wave" is associated with the " 10 points" of the Monas symbol.

The only problem with this arrangement is that the midpoint of the " 11 Wave" (in the 2-digit realm) is 49.5, which again falls at that obscure spot on the spine of the Monas Symbol. However, the nice part of this arrangement is that the " 4,7 , and 10 points" correspond with $\mathbf{3 3}, 66$, and 99 . Recall that these results are that "powerful way" of seeing the synchronicity between Metamorphosis and Consummata.

$$
\begin{aligned}
& 12+21=33 \\
& 24+42=+66 \\
& 72+27=\overline{99}
\end{aligned}
$$

If we shift the " 11 Wave" from " 10 points" to " 9 parts," the " 49.5 star" falls at the intersection of the Cross and the Sun Circle, making the same "octave plus 1" arrangement found earlier. Again, both ways of viewing the " 11 Wave" are valid.



In fact, if the " 11 Wave" was extended to " 10 times 11 " (or 110), the important midpoint would be 55. In this depiction, 55 is the "middle part" of the 9 parts of the Monas Symbol.

The number 55 is important in a special way. It's the "ultimate nave" of the Marshall's "Hitching sequence" of double-digit reversible primes discussed earlier(13 and 31), (17 and 71), (37 and 73), (79 and 97), (and the non-prime pair 39 and 93). Add all these together numbers and you'll find the average is 55 .

## One more correspondence.

We've seen that Dee used the 3 orientations of the Aries symbol to
 represent the numbers 3,6 and 9 . This suggests the " 3 groups of 3 " arrangement of the " 9 parts" of the Monas Symbol. His three orientations might (in a creative way) also be seen to represent the ( $+4,-4$, octave; null 9 " arrangement of these "9 parts" as well.
Here's how. Envision that single "null 9 part" at the bottom of the Monas Symbol instead of at the top. Down in that section, one finds the Aries symbol. Now, the "octave of parts" is above it.


In Dee's illustration, if we join the orientations that look like a rounded, capital letter "E" and the digit " 3 ," we get the shape of a "figure- 8 ". Enlarge the "figure- 8 " a bit, and now Dee's 3 orientations of the Aries symbol express the " 9 parts" as $(1+8=9)$.

Dee seems to have had this idea in mind. Otherwise he might have illustrated the three orientations of the Aries symbol in a "different order" that would have visually distin-
 guished each of the characters better.


But instead, Dee drew the "open part" of a letter "E" facing the "open part" of the digit 3 , hinting at a figure-8.. Knowing how cleverly Dee hid clues, this hardly seems accidental. It's curious how this, one of Dee's smallest illustrations, is so chock-full of meaning.

Instead of seeing this as $(1+8=9)$, it's perhaps more appriate to see it as the song goes, $(8+1=9)$.This can easily be done by using Dee's "inverted Monas Symbol." In fact, seems to be why he places it where he does in the chart (and why he leaves a gap in the Horizon line above it).


## Let's review this busy chapter

The Monas Symbol can be seen as a "measuring device" for the various "Waves" of Consummata. There are two ways to align these "transpalindromic octaves" of Consummata : either with the "points" or with the "parts" (like aligning with the inch marks" of a ruler or with the spaces between the "inch marks").

When the transpalindromic octaves are aligned with the " 10 points", wondrous correspondences with the 252 pretzel and the $4: 7$ ratio result.

When the "transpalindromic octave" are aligned with the " 9 parts," the "octave, null 9 rhythm" can be seen.
Besides dividing the " 9 parts" into " $8+1=9$," they can also be divided into "thirds."
This "thirding" might be seen as as " $3+3+3$," or even as (the spaces between 1 and 4), (the spaces between 4 and 7 ) and (the spaces between 7 and 10).

But instead consider them as their cumulative totals, 3,6 , and 9 .
This way its easy to see how the "oil 3,6 , and 9 " relates to the "water of $1,4,7$, and 10 ."

Review how 36, 63, 369,693 relate to the " 4 and 7 " in these charts and you'll more clearly understand why I depicted the Monas Symbol as doing an energized, gyrating, flamenco dance.


When a novice first views the Monas symbol, he might see an arrangement of geometric shapes or perhaps even a squatting one-eyed horned figure or even (as Dee points out) the astrological symbols of the planets.

When Dee or anyone "in the know" looks at the Monas symbol they saw ideas like 252, Metamorphosis numbers, Consummata, and the Exemplar Number all playing together and a harmonious symphony!

# "Straight lines" VERSUS "CURVY LINES" AND THE "3 STANZAS" 

Let's explore how Dee hints at these various groupings found in the Monas symbol:

$$
(1,4,7,10),(3,6,9) \text { and }(4+4+1=9)
$$

One of the smallest illustrations in the Monas is also one of the most important in the sense that it is the key that unlocks greater clues concealed in other illustrations.

In Theorem 21, Dee analyzes the various parts of the "inverted Monas Symbol." In his illustration, the part labeled B includes the "inverted Aries symbol." He then shows three alternative ways to see that "inverted Aries symbol."

Hoc denique annotare libuit,(animi recreandi gratios) Quòdipfum B,nobis, Rvsricas, tot literas expeditifsimè exhibet, quot Puncta,furfum, confpicienda in Capite, \& quafi Fronte gerit: iftas fcilicet tres: $m$ () vt\& alias quafi fex:(Súmatim auté ter tría)r v des valde \&impolitas,fluxiles volubilefq; : vt, ex Semicirculis, yno vel pluribus, eafdé effé factas videtis.
(1) With tips pointed downward, like a proper "upright" Aries symbol.
(2) With tips pointing to the right, like a rounded-off, capital letter E.
(3) With tips pointing left, like the numeral 3 .

Here, I've translated his comments about the illustration and organized them into 3 paragraphs to make their analysis easier. We've already explored how this passage refers to the "letters of the alphabet," but it carries an important "number" message as well.


Dee notes that the 3 shapes might "in a different way" be seen as 6 shapes (referring to six "half circles"). Then, he strangely says that 3 and 6 "Summed together make" 9. (Dee's original Latin words are "ter tria" meaning "thrice three" or " $3 \times 3$.")

Furthermore, his description of the shapes as "very ROUGH and in want of neatness, yet flowing and rolling" is quite important.

Dee's Latin word "fluxile" means "fluid or flowing, and over the years has been used to describe "a current air" or "the folds of drapery."

His Latin word "volubilis" means, "turning around, spinning, twirling, circling, or rolling"

To understand why this description is a huge clue, let's go back to one of Dees main themes, "coincidentia oppositorum," the "union of opposites."

To a like geometer like Dee, there's a big distinction between a straight line and the curved line. Straight lines are made with a tool like a straight edge (like the edge of a carpenter's square). Curved lines can be done freehand, but for neater, circular lines, a geometer's compass is used. Different tools for different results.

> It's the Sun A Moon. Its Fire A Water. It's Heaven A Earth. It's Hot $\&$ Cold. It's 12 \& 21. It's $144 \AA 441$. It's 1089 \& 9801. The list goes on.

Straight lines and curved lines are both We might summarize this as:
curvy or flowing f straight or rectilinear lines, they're just at different ends of the same spectrum, like "hot and cold" are both temperature or "wet and dry" are both degrees of humidity.

With this dichotomy in mind, let's take a fresh look at the Title Page. There's a huge visual contrast between the "straight" lines of the architecture and the "curvy" lines of the central emblem. Though the dome may be curved, the foundation, pedestals, columns, and entablature are all made from rigidly straight lines.

In stark contrast are the curvy lines of the "central emblem" which includes the egg shape, the shield, the draping grapevines, and in particular of the "flowing ribbons."

In fact, Dee's description of the " 3 orientations of the Aries symbol,"very ROUGH and in want of neatness, yet flowing and rolling" is an apt description of these ribbons. They are most definitely "flowing and rolling" as if being tossed about by a breeze.

They might not seem "ROUGH" but Dee's original Latin word "RUDE," also translates as "unformed, unpolished, awkward, rude, clumsy", or even "wild"

(Lewis and Short, p. 1603)
The flowing ribbons are "awkward" and "in want of neatness" in the sense that they don't fit inside the between the columns very well. Not only are they visually "clipped off" by the protruding bases of the columns, the left-hand ribbon is also snipped off by the upper part of the left column.


As we've seen, raising the emblem upwards to its "restored" position in the "theater" is a clue to the hidden geometry involved in the "angles of the Mercuries' spears." That certainly solves the Title Page's "in want of neatness" problem.

Remember, Dee calls, "the 3 orientations of the Aries symbol" "very ROUGH and in want of neatness, yet flowing and rolling" but also they represent " $3,6,9$."

Likewise the ribbons on the Title Page are, "very Rough and in want of neatness, yet flowing and rolling."

Using Dee's "syllogistic" logic, the flowing ribbons might represent " $3,6,9$ " as well!

If I asked third-graders to find the "hidden 3,6 , and 9 " in the "flowing ribbons," they would find them in a snap.

The mid-section of the righthand ribbon wiggles in the shape of a figure-3. The bottom of that ribbon makes to curvy figure-9. Hold the book upside down and the nine becomes a six. The same numbers, $3,6,9$, can also be seen on the other ribbon, only backwards.


$$
\begin{aligned}
& 12+21=33 \\
& 24+42=+66 \\
& 72+27=\frac{+}{99}
\end{aligned}
$$

With just a little imagination, one might even see the numbers 33,66 , and 99 . These are key members of that "powerful display of the synchrony between Consummata and Metamorphosis."

Nowadays, the digits 6 and 9 are often drawn with the "strait stems," but most typefaces still use curved stems. More importantly, in the 1500 's, the digits 6 and 9 had curvy stems, whether and written or set in a typeface. (Dee used curvy-stemmed 6's and 9's in his handwritten "Library Catalog" of 1583.

I had previously used the metaphor of "the oil of $3,6,9$ " versus "the water of " 1,4 , 7,10." Let me revise this metaphor to numerals "formed with "curvy, flowing lines" versus those formed with 'straight lines". (The numeral 10 does has a curvy line because it has a " 0 " in it, but to Dee, 10 was essentially a "return to 1 ", making that " 0 " somewhat superfluous.)

In the "Thus the World Was Created" chart, Dee depicts the number 4 with a triangular top, using only a vertical, a horizontal, and diagonal line. He draws the number 7 using only a horizontal line and a diagonal line.


In summary, Dee's concepts of "very ROUGH and in want of neatness, yet flowing and rolling" and the numerals " $3,6,9$ " connects these two illustrations.

So, like good detectives, let's pick up the clue of the "maxim" written on the "flowing ribbon," then look for more evidence of "very ROUGH... yet flowing..." things elsewhere in the Monas.

I described earlier how this maxim is a "musical round" having has three stanzas which can be summarized as $(7+1=8),(8+1+=9)$, and $(9+1=10)$. Here is a visual summary of the three stanzas.


## The story of $7+1=8$

To explore the "story of $7+1=8$," let's first return to the " 3 orientations of the Aries symbol." Because the illustration is an engraving (ink in a grooved plate), it was printed on a separate pass through the printing press than all the letterpress type (ink on raised letters).

Here, I separated the material which the two "passes" would have printed. It doesn't matter which process was printed first. What's important is the idea that they are separate.


Next, I've separated the "engraving" pass from the "letterpress" pass of the Artificial Quaternary.


This similarity prompted Monas translator C.H. Josten to write in a footnote, "Does Dee conceive these shapes as brackets? They appear in all subsequent editions as swung brackets." Translator J.W. Hamilton-Jones actually re-illustrated them as very curly brackets instead of pairs of half circles (unfortunately depriving readers of a valuable clue).

I have previously referred to this as Dee's "unexplained division" of "8 into 1 and 7", and " 7 into 4 and 3 ." The fact that Dee incorporated these numbers in his Artificial Quaternary is testimony to their importance.


Hark, I heareth music playing: Dee seems to be seen singing the first stanza of the "musical round of the flowing ribbons," $(7+1=8)$.

But, Dee also divides the 7 into 4 and 3. This is a succinct depiction of the theme he wrote about in Theorems 6 and 20: the Cross of the Elements can be seen as Ternary (two points and a line) or as Quaternary (4 lines), which add to the "Most Excellent Septenary."


Dee sees 7 as " $3+4$." He also sees 7 as a vey close relative of 8 . In short, from " 8 " Dee derives " 1,7 , 4, and 3."

With the exception of 10 , (which is a "return to $1,{ }^{\prime}$ ) Dee has presented us with all the digits necessary to describe the Symmetry of the Decad, seen either as "parts"(thrice 3 ) or points( $1,4,7,10$ ).

Furthermore, his dividing the " 7 into 3 and 4 " suggests that important " $3: 4$ ratio," which can be expressed as the two fractions $3 / 7$ and $4 / 7$.


We've explored howwell-integrated $4 / 7$ is with the "four steps":

$$
\frac{4}{7}=\frac{12}{21}=\frac{24}{42}=\frac{36}{63}=\frac{48}{84}
$$

And how well-connected it is with the 99 Wave: $(4 \times 99=396)$ and ( $7 \times 99=693$ )

$$
\frac{4}{7}=\frac{396}{693}
$$

Next let's look at the "engraving" pass and the "letterpress" past of the "Artificial Quaternary chart." There are only 4 "engraved" brackets compared with 12 "letterpress" brackets.


The information that these engraved brackets are associated with include the "numbers 4 and 3" as well as the jumbled letters of "Quaternary Rests in the Ternary." We also find reference to the Tenness (which we were looking for a moment ago).

And there is also a reference to the " 4 steps" $\left(1^{\circ}, 2^{\circ}, 3^{\circ}, 4^{\circ}\right.$, Gradu. $)$.
Let's assemble all the information associated with these "very ROUGH and in want of neatness, yet flowing and rolling" things including the "first stanza" of the "flowing ribbons." It tells quite a story:


But this story does not involve " 8 " very much. That's because 8 is the Queen of the next stanza which sings the tune of $(8+1=9)$.

## The story of $8+1=9$

Dee's intended meaning for $(8+1=9)$ is pretty obvious. It's the "octave, null 9 " rhythm which he perceived in the Base 10 numbering system.

We've already seen how the single digits, the 9 Wave and the 99 Wave relate to the 9 parts of the Monas Symbol. The octave is like the "two circles," which is the general plan of the "ballooned- 360 Thus the World Was Created" chart and the John Dee Tower. Oh, and let's not forget the most obvious: "two circles" is also a pretty good description of the numeral 8 itself.


This "octave, null nine" story is a prominent part of one of Dee's other illustrations, part of which has that "very ROUGH and in want of neatness, yet flowing and rolling" characteristic: the "Thus the World Was Created" chart. Here, I have divided this chart into its "engraving" pass and its "letterpress" pass.


The 4 larbe,curved brackets on the right side of the chart are quite "flowing and rolling." They are a little "ROUGH and in want of neatness" in the sense that mathematically speaking, the largest arch yearns to be "ballooned" into a full half circle.

Note that the "engraving" pass includes the digits " 1 through 7," the digits " 1 through 8 ," the words Metamorphosis and Consummata, various Monas Symbols, and of course that infamous clue "the Engraved 2."

It also includes many of the lines which enclose those digits. (On close inspection,the letterpress lines can be seen to be comprised of numerous short lines of typesetting and are rather thin. The engraved lines are continuous and somewhat thicker.)


One way to see the " $8+1=9$ " story here is the "octave of digits with the HORIZON AETERNITATIS (9) above it."

But another way is to just see the whole left-hand column of the chart as an "octave measuring scale," topped off by the "null 9 HORIZON."

Now, if we slide to the "flowing and rolling" curved arcs over a bit, they become very "bracket-like."

The two "Below" arcs bracket (1, 2, 3, 4).
The SUPERCELESTIAL arc brackets (5, 6, 7, 8).
The "SIC FACTUS EST MUNDUS" arc "brackets" the whole octave.


The "flowing and rolling" arches might be seen as "mearuring brackets"

Let's visualize this as an octave of Metamorphosis numbers. This is when that "Engraved 2" comes in handy. Multiply that "Engraved 2" times 6126120 and the Exemplar Number, 12252240 , is attained.


To summarize, this $(8+1=9)$ stanza is about an "octave of parts" plus "another part" making the full 9 parts of the Monas Symbol. This stanza doesn't really concern itself with the "10 points" of the Monas Symbol. That's what the next stanza is about.

## The story of $9+1=10$

The third stanza, $(9+1=10)$, highlights the relationship between 9 and its neighbor 10 . Ths duo make an odd couple, 9 being a "null number," and 10 being a "return to one."

Ten is important because it is our Base Number, and 9 is its transpalindromizer.
The transpalindromizer is always 1 less than the Base number.
Another way of expressing this is (the transpalindromizer $+1=$ Base Number) So, in Base Ten, that tune is $(9+1=10)$.

Dee celebrates the relationship between 9 and 10 with his "measuring stick," the Monas Symbol. It has 10 points on its spine, or 9 parts between those points.

The Symmetry of the Decad $(1,4,7,10)$ divides those 9 parts into 3 groups of 3 . The first group contains 3 parts. The second group contains 3 parts as well, so cumulatively in both groups there are 6 parts. The third group as 3 more parts, so cumulatively there are 9 .

What does this remind you of?
It's the same tale Dee told about the
" 3 orientations of the Aries symbol"
"The 3, clearly illustrated here, in a different way are 6,
(which Summed together makes $3 \times 3$ )."


Let's return to the "printing analysis" of the "Thus the World Was Created" chart, this time focusing on several aspects of the "Below" section of the "engraving" pass.


As we've seen, the numerals 1,4 , and 7 are enlarged and are listed in their own special column, hinting at the Symmetry of the Decad. Note that the boxes enclosing these numerals are "engraved" lines. But similar "engraved" lines also surround the Lunar Mercury Planets Symbol and the Solar Mercury Planets Symbol (the full Monas Symbol), which I have explained represent " 8 " and " 9 " respectively.

> Allow me to slide Dee's phrase "Ancient enigma of the Symmetry of the Decad explained" to the right a bit.


Now the phrase seems to be describing the digits $1,2,3,4,5,6,7,8$, and 9 , which can be seen as the " 9 parts" of the Monas Symbol. (Seeing it as " 9 parts" is helpful when visualizing dividing the Monas Symbol into thirds. Dividing 10 by 3 will lead you down the wrong road, where you won't come across the " $1,4,7,10$ " Symmetry of the Decad.)

Remember that Dee hints about this arrangement in Theorem 23. After geometrically constructing the Monas Symbol with its 10 points, he adds that it also consists of "nine equal parts" and that the "Mechanicum...may go about performing his work in another way."

Note that the numbers 3,4 , and 7 are key players in this game.
The 4 and 7 are the "pivot points" between 1 and 10 .
The points (1 through 4), (4 through 7), and (7 through 10), each have gaps of " 3 parts."
These key numbers, 4,3 , and 7 hearken back to where this song started... back to the first stanza of the musical round, $(7+1$ $=8)$, about which Dee notes that $(7=4+3)$. The song begins all over again, and continues looping, going 'round and 'round...


To summarize, Dee uses the graphic distinction between the opposites of "curvy, flowing" and "straight, rectilinear" to hint at the connections between his illustrations.

He also uses the graphic distinction between the "engraving" pass through the press and the "letterpress" pass through the press to further hint at these interconnections.

These 3 simple addition equations reveal marvelous things about how number works. It's no wonder Dee displayed them them so prominently (though quite cryptically) on the Title Page.

The main thrust of all this is that while each of these three stanzas tells a different story, the three stories are all related.


Following the lead of Dee's "unexplained division" of " 8 ", (the additive result of the Artificial Quaternary), we might envision 9 and 10 be divided up in the following ways:


It seems so clever of Dee to have used the "curvy line/straight line" dichotomy as a clue in his illustrations. However, it seems like something he noticed rather than made up. Perhaps by going into great detail about the shape of Roman numeral letters being geometrically similar to the Cross (like L,V and X ,) he is suggesting the reader look for similar graphic characteristics in the Arabic numerals.

It seems as though Dee wanted to help the reader to recognize the ultimate "curvy/ straight" dicotomy: the oppositeness of zero-one, the fountainhead of retrocity.
" $O$ " is curvy. " 1 " is straight. It doesn't get much simpler. Just as Dee says in the very first Theorem, "The very first and most simple representation...of things..is made by means of a straight line and a circle."

No doubt Dee saw how the "curvy/straight" dichotomy naturally appears in what I call the "powerful display of synchrony between Consummata and Metamorphosis."

The curvy 3's, 6's, and 9's are obvious in the 33,66 , and 99.

$$
\begin{aligned}
& 12+21=33 \\
& 24+42=+66 \\
& 72+27=\underline{99}
\end{aligned}
$$

But hidden on the other side of these equations are the "straight lined" 1,4 , and 7. Curiously, all of the numbers on the left side of this display contain the numeral 2.

Allow me to remove that "common 2 " for a moment. What we're left with is 1,4 , and 7 , all "straight line"digits (as well as key digits in the Symmetry of the Decad).

Perhaps you can explain why it works this works this way. Remember, I've done some "strange" mathematical things here. Starting with the first 3 Metamorphosis numbers, I found their "reflective mates"." Besides arbitrarily deleting all the " 2 's, I have ignored place values (for example, with 21, 24 and 27 , I've actually taken away 20 , not 2.)


In the next chapter, we'll see how Dee uses the "curvy line/straight line"dichotomy as a huge clue that helps weave the Monas Heiroglyphica together.

# What does the first word of <br> DeE's song, "MERCURY" <br> HAVE TO DO WITH LUNAR AND SOLAR MERCURY? 

Knowing that Dee's song goes $(7+1=8),(8+1=9),(9+1=10)$ makes it easier to understand what he's talking about in Theorems 12, 13, 14, and 17, which explain these "planetary" diagrams:


Dee divides the planet Mercury into two flavors, Lunar Mercury and Solar Mercury.


In Theorem 12, he introduces Lunar Mercury. He discusses the " 4 revolutions" (Saturn, Jupiter, Moon, and Lunar Mercury) and summarizes the group with the Lunar Mercury Planets Symbol. (Lunar Mercury, with Aries "legs" added)


In Theorem 13, he introduces Solar Mercury adding that it's the "Uterine Brother" of Lunar Mercury. In the text, Dee suggests that Mars, Venus, Sun, and Solar Mercury are all Solar Planets. One might assume that these 4 would group together just like the Lunary Planets, but Dee does not make such an illustration.


Having "4 Lunary things" and "4 Solary things" sure would nicely relate to the "octave" rhythm found in number, but this is not how Dee proceeds.

Instead he introduces the "Total Inferior Astronomy" diagram in which the Sun and Solar Mercury are both represented by the number 7.

One might think that this "Cross on a 5 x 5 grid" would be a perfect way to display be " +4 , -4 , octave; null, 9 " rhythm of number, by showing 4 Lunary Planets, 4 Solary Planets and ninth "null" rectangle in the center.

But Dee doesn't take this "easy octave" route. He does utilize a "cross shape" of 8 grid squares with a shaded grid square in the center, but the actual planetary symbols are placed in a much different order. The number 8 doesn't appear at all. Instead the Sun and Solar Mercury are both represented by the number 7 .

Why does he do that?


The short answer is "so the full " $(7+1=8),(8+1=9),(9+10=10)$ " song can be sung." To understand what this means, let's take it one step at a time.

The Spiral diagram of Theorem 18 provides the next clue. At first glance, it looks like 7 spirals with 7 planets, but closer inspection tells a different story. The labeling digits (1, $2,3,4,5,6,7$, and even the " X ") are associated with points on the spiraling black line.

But the 7 planetary symbols are associated with the spaces between successive revolutions of that black line. This is reminiscent of the two ways of looking at Monas symbol: "parts versus points."


Dee's shape does not to spiral endlessly. Instead, its outermost spiral converges back onto itself ultimately "contacting at a point" Dee simply calls "X."

The most obvious decipherment is that the " $X$ " represents the Roman numeral 10. However, based on the pattern of the digits on the spiral, it would more appropriately represent the number $\mathbf{8}$. In other words, the spiral diagram seems to be singing the $(7+1=8)$ stanza of the musical round.


Spirals can be dizzying to look at, so let's transfer the Spiral's stanza $(7+1=8)$ to the vertical spine of the "Monas Symbol measuring stick."

This is all very nice, but there are two "parts" at the top of the Monas Symbol, which are as yet unaccounted for (parts 8 and 9).


Dee provides a clue to these missing pieces in the short Theorem 14 saying "this whole Magistry" depends upon the Sun which he sees as the "Father" and the Moon which he sees as the Mother." This relates to the "parents" that "Mercury becomes" in the "maxim on the flowing ribbons."

But Dee's real intent is left ambiguous until he clarifies it (cryptically) in the "Thus the World Was Created" chart at the tail end of the Monas in Theorem 23.

In the "Below" half of this busy chart, he separates Solary things (Fire and Air), (Red and Yellow alchemical stages), and the (Aetheric Celestial bracketing arc) from Lunary things (Water and Earth), (White and Black alchemical stages), and (the Terrestrial bracketing arc).


All the "Lunary things" are associated with a Lunar Mercury Planets Symbol. All the "Solary things" are associated with the Solary Mercury Planets Symbol.

As this "Below" half already lists the digits $1,2,3,4,5,6,7$, it appears as though these two symbols represent $\mathbf{8}$ and 9 .

Look how nicely they complete the "nine parts" of the Monas Symbol measuring stick.


The Lunar Mercury Planets Symbol is a combination of the Aries symbol, the Cross of the Elements, and the Lunar crescent.

The Solar Mercury Planets Symbol incorporates these same three symbols plus a "Sun Circle." The center of the Sun Circle is "one" point. As Dee Says in Theorem 2 "things related to the circumference of a circle" cannot exist without the Service of the Central Point."


So in this sense, the Lunar Mercury Planets Symbol plus a "point," makes the Solar Mercury Planets Symbol.

Recognize this?
It's the second stanza of Dee's song,

$$
(8+1=9) .
$$



As the Sun and Moon are opposites, like Father and Mother are opposites, like Solar Mercury Planets and Lunar Mercury Planets are opposites, Dee seems to be is inferring that 8 and 9 are opposites.

They are definitely not transpalindromic mates, but in the "octave, null 9" rhythm of number, they can definitely be viewed as antithetical or complementary. Without the octave, the 9 wouldn't have an opportunity to be "null." Without the null 9 , the octaves wouldn't be properly separated.

We've sung two stanzas so far, what about the third stanza, $(9+1=10)$ ?
In our earlier analysis of this song, the $(9+1$ $=10)$ stanza related to the " 9 parts and the 10 points" of the Monas Symbol.

In comparing the various number Waves ( 9 Wave, 99 Wave, 1089 Wave, 10890 Wave, ...) to the "Monas Symbol measuring stick," a nave was born at 5445 in the 4 -digit realm of numbers. This "shift" suggested that various Waves might be associated with either "parts" or "points."

So let's associate Dee's 9 planetary symbols with 9 of the points of the Monas Symbol.


Remember how confusing it was that Dee represented both the Sun and Solar Mercury with the number 7 in the "Total Inferior Astronomy" diagram?

Suddenly his reasoning becomes clear.
In this diagram,
Lunar Mercury is associated with point 4, and Solar Mercury is associated with point 7.

The two kinds of Mercury are aligned with the pivot points in the Symmetry of the Decad.

As Dee calls Lunar Mercury and Solary Mercury
"Uterine Brothers,"
this means that 4 and 7
are "Uterine Brothers."


Let's return to the alignment of the " 9 Wave" with the points of the Monas Symbol. Now we can see that 36 and 63 are "Uterine Brothers."

This suggests that any numbers in the ratio 4:7 are "Uterine Brothers," for example the other 3 of the " 4 steps," (12 and 21), (24 and 42), (48 and 84).

Recall that the " 12 and 21 pretzel" involves this little 4:7 ratio, as 108: 252, and 252:441 are in the 4:7 ratio. Recall that in the "extended pretzel," 396: 693 are in the 4:7 ratio.

This can be seen by putting the 99 Wave on this "Monas Symbol measuring stick." Thus 396 and 693 are also Uterine Brothers!

And, of course, in the midst of this analysis, we are comparing "curvy line" digits $(36,63,396,693)$ with "straight line" digits $(4,7)$.



The only thing unaccounted for here is that "point 10 ." To Dee," 10 was a return to $1 . "$ The most graphic way to see this is in the "triangular representation" of the Symmetry of the Decad.

Another way to see " 10 is a return to 1 " in the display of what I call the "Symmetry of the Centad." where the symmetry pivots around 40 and 70.

This is what Dee means when he includes "Denaria, which is, 1, 10, 100, to Infinity" in the Artificial Quaternary chart.

In Theorem 16, after several mathematical gyrations, Dee ends up with the "CENTURIO" (number 100). He also says that enumeration proceeds "Unum, Decem, Centum" $(1,10,100)$.


That "point 10 " is not insignificant at all.
It is the King.
Just as "10 is a return to 1 ," the King is the sole ruler of the realm.

But just as the King cannot be king without subjects, this third stanza, $(9+1=10)$, brings us to 10 , whose pivot points are 4 and 7 (key subjects in the King's realm of single digit numbers).

This harkens back to the first stanza in which we had Lunary things representing 4 digits and Solary things representing 3 digits, bringing us up to 7 . The song starts over again, and goes 'round and 'round.

To recap, Dee's "song" regarding
"Mercury... becoming parent, king to all the planets," is a giant metaphor for understanding how number works.

He uses the various planets or grouping of planets to extend his numerical metaphor.

The first stanza $(7+1=8)$ tells the story of the $3: 4$ ratio (and the $3 / 7$ and $4 / 7$ fractions).
The second stanza, $(8+1=9)$ tells the story of the "octave, null 9."
The third stanza $(9+1=10)$ tells the story of how there are 9 parts between 10 points.
The 10 points is the Symmetry of the Decad with its pivot points at 4 and 7 ,
which relate back to the first stanza.
The song rolls on.

Dee is nice enough to provide us with an illustration that sings a full "round" of all 3 stanzas: the "EGG" diagram of Theorem 18. Its principal shapes are much more "flowing and rolling" than they are "straight and rectilinear," thus connecting it to the other illustrations which provide clues about the "song."

We've previously explored how the
Lunar Mercury Planets in the egg white represents " 8 ,"
the Solar Mercury Planets in the yolk represent " 9 ," and the whole shell represents " 10 ."

Sound familiar?


These numbers, " 8,9 , and 10 ," are the 3 "results" of the 3 stanzas, $(7+1=8),(8+1=9)$, and $(9+1=10)$.

In other words, the "EGG" diagram sings all 3 stanzas!
This understanding sheds new light on Dee's remarks about the this diagram:

> "As we were contemplating
> both the Theoretical and the Heavenly motions of that Celestial MESSENGER [Mercury], we were taught that the figure of an EGG might be applied to these COORDINATIONS."

After quizzing the reader on what the egg white, yolk, and shell represent, he adds (in all capital letters):
"HERE WE HAVE PRACTICALLY THE WHOLE, IN NATURE'S PROPORTIONS."
(Dee, Theorem 18, p. 17 verso)
The "WHOLE" one egg incorporates the key "PROPORTIONS" of (4:7), the (8, null 9) rhythm of number, and the $(1,4,7,10)$ Symmetry of the Decad.

Dee describes the "Spiral"diagram as a "rolling, and rolling some more" of shell, white, and yolk. As this recipe includes all of the 3 parts of the egg, we should expext to see 8,9 , and 10 in the swirling mass. The clue is the X .

In the text of the Monas, Dee makes a big deal out of the idea the X is the Roman numeral for 10 .

Let's take a fresh look at the diagram in which Dee put an $X$, where an " 8 " seemed more appropriate. It's almost as if he "left out" 2 additonal cycles that woud include 8 and 9 .


Seen this way, the EGG and Spiral diagrams sing not just the song of " $(7+1=8),(8+1+9),(9+1=10)$," but the fuller song of $1,2,3,4,5,6,7,8,9$, and " 10 which brings you back to 1 ."

With this in mind, we can look at the Title Page with fresh eyes. Three things, the central Monas Symbol, the Egg that surrounds it, and the "flowing ribbons" each sing the 3 stanzas of Dee's catchy numerical tune.

If Dee had simply found 4 Lunary Planets and 4 Solary Planets to make a nice octave, this "more thorough" story would not have been able to be told. (Besides, in the Dee's time there were only 7 known "planets.")


The "song of the flowing ribbons" reverberates through all of the Theorems of the Monas chanting about the harmony of number in a grand symphony.

One can hear the rhythms of Consummata and Metamorphosis in the music. The $(8+1=$ 9) stanza is particularly recognizable as Consummata. The other 2 stanzas incorporate the idea of 4:7, which is equivalent to $12: 21$ or $24: 42$, thus integrating the "pretzel" as well as the first 2

Metamorphosis numbers.

Another way to see Consummata and Metamorphosis is in the Lunary Planets, (Saturn, Jupiter, Moon, Lunar Mercury) or (1, 2, 3, 4). In Dee's Artificial Quaternary, the "essence" of (1, $2,3,4)$ is $(1,2,3,2)$ which "results" in $\mathbf{1 2 , 8}$, and $\mathbf{2 4}$.

The numbers of the Solary Planets (Mars, Venus, Sun or Solar Mercury) or (5, 6, 7). In the progression of "essences" we found that 6 "requires nothing" because 2 and 3 already existed. This leaves ( 5 and 7 ) which sum to 12, the first Metamorphosis number.

# PRIME NUMBERS AND THE "Hitching Sequence" 



If you look closely enough, this diamondshaped chart of 1-digit and 2-digit numbers has wondrous stories to tell.

Along the upper left edge are the single digit primes, $2,3,5$, and 7 .

The 2-digit primes appear to be scattered randomly, but as all primes are odd (except for the number 2), they all appear in rows whose members end in $1,3,7$, or 9 .


Because we are using this dandy diamondshaped chart, which has "perfect reflective symmetry," we can easily identify those primes whose reflective mates are also primes. There are four pairs of them scattered about the chart (13 and 31), (17 and 71), (37 and 73), (79 and 97).

Marshall calls these "reversible primes" and are part of what he calls the "Hitching Sequence." (What they "hitch" will become clear later in this chapter.)

To simplify the picture, let's connect each of these pairs of reflective mates with a line.

The midpoint of the lines show the various "averages" of these four pairs of "reversible primes."

The average of 13 and 31 is 22 . The average of 17 and 71 is 44 . The average of 37 and 73 is $\mathbf{5 5}$. The average of 79 and 97 is 88 .

Some of their "averages", 22, 44, 88 are an interesting progression of "doublings," but the most important of these "averages" is actually $\mathbf{5 5}$. Here's why:


If we connect 13 and 97 , their average is 55. If we connect 31 and 79 , their average is also $\mathbf{5 5}$. And, of course, the average of 37 and 73 is $\mathbf{5 5}$.

But 17 and 71 got left out in the rain. To include them, Marshall discovered there was another important pair of "reflective mates" which needed to be included in this picture: $\mathbf{3 9}$ and 93.

This new pair work symmetrically with 17 and 71 providing two more connecting lines cross at that nexus number 55.

Wait a minute. It seems unfair to arbitrarily pop 2 composite numbers into this "picture of primeness," just to make it work. Hovever, 39 and 93 aren't just any random composite numbers.

The only factors of 39 are 3 and $\mathbf{1 3}$. The only factors of 93 are 3 and $\mathbf{3 1}$. As you can see, 13 and $\mathbf{3 1}$ are one of the 4 pairs of "reversible times." Thus, 39 and 93 are simply 13 and 31 on steroids.

Just observing the beautiful symmetry of these crisscrossing lines will help you see that 39 and 93 should be included in the "Hitching sequence."


More supporting evidence can be seen by looking at the individual digits that make up these 2 -digit numbers. What I mean is that $(13,31),(17,71),(37,73)$, and $(79,97)$ are all made from the digits $1,3,7$ or 9 . Note that the newcomers, 39 and 93 , also pass this test.

In summary, the Hitching Sequence is made from "4 pairs of reversible primes" and "1 pair of reversible composites." That's just the way it is.

The important thing to note about this symmetrical asterisk of intersecting lines is that it is not centered in the overall diamondshape. It's nexus point or "nave" is "shifted" downwards from 49.5 to 55 .


Does this remind you of something? Recall how the 4-digit range of Base 10, a "shift" occurred where suddenly the "nave" of 5445 appeared. These two "shifts" are actually both expressions of the same "shift," as $55 \times 99=5445$.

What this shows is that primes (the reversible primes, also including 39 and 93) are related to Consummata (the 9 Wave, the 99 Wave, 1089 Wave,...).

And, as we saw in a previous chapter, the Metamorphosis numbers arrange the primes in perfect symmetry.

This helps explain the name "Hitching sequence." Since it is connected to Consummata and Metamorphisis, in a sense it hitches them together.

In short, the 1-digit and 2-digit number realm as two naves. One nave is 49.5 , which is half of $99(9 \times 11)$. The other nave is 55 , which is half of $110(10 \times 11)$.

Why " 2 naves"? To better answer this question, let's study the "opposites" of the primes, the composite numbers.

## Composites and the fascinating ( $1+9+9+9 \ldots$ sequence)



Obviously, this chart of composites is exactly the opposite of the chart of primes (except for 1 which is its own special case). But this doesn't tell us much. Let's analyze it more deeply by looking at the charts of the "multiples of $\mathbf{1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9}$, and 10," shown on the next page.

The patterns tell fascinating stories. The chart for " 2 " is a swath of diagonal stripes. The chart of " 4 " is those basically those same stripes, only "checker boarded." The chart for " 8 " the further minimalizes that "swath of diagonal stripes."

Note how the charts for 5 and 10 are similar to each other, and how the chart for 7 has its own eclectic persona.

Most pertinent to this study are the charts for those "flowing and rolling" numbers " 3,6 , and 9." The chart for " 9 " is obviously the " 9 Wave," the widest horizontal band in the chart.

Note that the " 9 " Wave is also included in the "chart for " 3 ," which is a swath of horizontal bands. The chart for 6 is kind of a "happy medium" between the charts for " 3 " and " 9 ." It includes "every other" member of the chart of 3 , some of which are members of the 9 Wave.


## What does all this have to do with the Monas symbol?

It seems as though the " 9 parts and be 10 points" of the Monas symbol should be associated with the chart of the "multiples of $\mathbf{9}$ " and the chart of the "multiples of $\mathbf{1 0}$," like this:


In the case of " $\mathbf{2}$ Monas symbols," the charts seemto indicate indicate that there should be a total of" 18 parts and 20 points."


For "3 Monas symbols," the charts indicate that there should be of "27 parts and 30 points."


But there's something wrong with this procedure.
The gap between "parts and points" is getting wider and wider.
That beautiful "Symmetry of the Decad" which the Monas symbol exhibits is being swept down the drain and must be rescued!

Remember, the "Symmetry of the Decad" was reallyfound by dividing the 9 (parts) by 3 , not the by dividing the 10 (points) by 3 . This is obvious because 9 divided by 3 makes a whole number (3), but 10 divided by 3 makes an endlessly-repeating decimal which is not a whole number (3.333...).

This apparent conflict is resolved when the 10 points are arranged in a triangle in which" 10 is a return to $1 . "$ As point 1 and point 10 are the same point, the 3 sides of the triangle each have 3 parts. The "parts" and "points" are happily singing the same song.


Similarly, "3 Monas symbols" have a total of 27 parts. Seen as a triangle, this means 9 parts per side. This arrangement only requires 28 points. Point 29 and point 30 have become superfluous.

We could go on, but the pattern is clear. To maintain symmetry, the number of points should always be "one more" than the number of parts.
(Or as Dee puts it, "made perfect" by the addition of a sharp, stable point." That point might be seen as that single point which is located at the summit of these sharp-peaked, yet extremely stable equilateral triangles.)


Next, the " $\mathbf{2}$ Monas symbols" have a total of 18 parts. Seen as a triangle, this means 6 parts per side. All this is accomplished with only 19 points. That 20th point is not needed.



An effective, graphic way to depict what's going on here is to "stack" the Monas symbols so that they "share" points"

In the stack of " 2 Monas symbols" you can see that 18 parts are still able to be divided into thirds.

Dee seems to hint at this arrangement and Theorem 21, where he says the "MEMBERS" (Moon Sun, Cross and Aries) of the Monas have a "vigorous MAGNETIC virtue." If these "MEMBERS" click together magnetically, it means that the whole Monas symbol is a magnet. In this depiction, two "Monas symbol magnets" are clicking and sharing a common point.


In a stack of "3 Monas symbols," the 27 parts still divide nicely into thirds.

But when we look at what I call a "Monas symbol totem pole" (a stack of 11 symbols), what's going on here becomes clearer.

On the left side of the totem pole are the "multiples of 9 ," which is the " 9 Wave." For simplicity, I'll call it the "9 +9 +9... sequence."

On the right side of the totem pole, the parts can be boiled down to what I call the " $\mathbf{1}$ +9 +9 +9... sequence."

While Dee never mentions the idea of "multiple Monas symbols," his use of an upright and inverted versions suggests we think creatively about the symbol.

Just as the Artificial Quaternary is a "starter clue" pointing to the pathway to 2520 ( 1 $\times 2 \times 3 \times 2 \times 2 \times 3 \times 5 \times 7=2520), 9+1=10$ is a "starter clue" leading to an understanding of these two

$$
\begin{aligned}
& \text { Monas symbol } \\
& \text { totem pole }
\end{aligned}
$$

 great sequences.

Here are the numbers of the " $9+9+9$...sequence" on left side of the "totem pole.

Here are the numbers
of the " $1+9+9+9 \ldots$...sequence" on the right side of the "totem pole."


Here's a snapshot of the 2 sequences interacting. I've depicted them in a "peanut-shape" because they're related like two peanuts in a shell. the first "peanut "is "zero-one," the root of all the retrocity that ensues.

Its energy can be seen in the next peanut, the 9 parts and 10 points of Dee's Monas symbol. There's also an interconnection between "18 and 19," "27 and 28 ,".. all the way up to the electric zing between " 99 and 100."

Just as there is a 100 at the top of the "totem pole," I've included 100. Even thought its a 3-digit number, its important to see in this pattern because 100 is a "return to 10 ," which is a "return to 1 ." Further out all the multiples of $10,(1000,10,000,100,000$, a million... ) are all in the " $1+9+9+9 \ldots$ Wave." This gives you a good sense of why the Greeks considered 9 "The Horizon Number."


Notice that both of these sequences are made up of transpalindromic mates. However, one important distinction is that the " $1+9+9+9 \ldots$ sequence" has a whole number "nave" (55), where the " $9+9+9 \ldots$ sequence" does not.


The "shift" between these two sequences is intrinsically related to the "shift" we saw in the rhythm of Base 10. In the 4-digit range of number, suddenly that whole number "nave" of 5445 appeared (as $55 \times 99=5445$ ).


The symmetrical centerpoints of these sequences now become more apparent. The fulcrum of the 9 Wave is in the exact center of the chart (at 49.5). The center of the $(1+9+9+9 \ldots$ sequence) is 55 .

But there's something else you might recall about 55. It's the nave of the Hitching sequence! In the 2-digit range of number, the 4 "reversible primes" and the 1 pair of "reversible composites" all average out to 55. (Mathematically they sum to 550, which, divided by 10 , makes 55.) Geometrically, tht nave of 55 is the nexus of all the crisscrossing lines explored earlier.

Notice that "37 and 73," a pair of reversible primes from the Hitching Sequence, are proud members of the " $1+9$
 $+9+9 \ldots$ sequence."

The Hitching sequence is all about primes.
The ( $1+9+9+9 \ldots$ sequence) doesn't appear to have anything to do with primes.
Yet both of these sequences share a the same nave.
This hints at an inherent orderliness among the prime numbers.
To get a clearer picture of what's happening here, let's divide each of the "Monas symbols in the totem pole" into thirds, and study their parts and points.


The left side of the totem pole is the "multiples of 3 " sequence or the " $3+\mathbf{3 + 3} \ldots$ sequence".

The right side
of the totem pole is the " $1+3+3+3 \ldots$ sequence".


Here's a picture of what these two sequences look like combined on a diamond-shaped chart. Notice that the longest horizontal rows here are the " $9+9+9 \ldots$ sequence" and the " $1+9+9$ $+9 \ldots$ sequence."

Of course, the real "pattern-setter" here is the first pair at the top of the chart (or the bottom of the totem pole) where " 0 and $1 "$ are in the same peanut.

The next "peanut" pair is 3 and 4. This is Dee's famed "Quaternary Rests in the Ternary."

In each of the "peanut" pairs of this chart, one member is always "one more" than its partner. This is exactly the song that Dee sings on the "flowing ribbons" of the Title Page:

$$
(7+1=8),(8+1=9), \text { and }(9+1=10) .
$$

(The " $9+1=10$ " being the "parts to points" of the Monas symbol)

There are a lot of peanuts swirling around this $(3+3+3 \ldots$ sequence, $1+3+3+3 \ldots$ sequence $)$ chart.
There are considerably fewer in the $(9+9+9 \ldots$ sequence, $1+9+9+9 \ldots$ sequence $)$ chart.
For a happy medium, let's look at the ( $6+6+6 \ldots$ sequence, $1+6+6+6 \ldots$ sequence) chart.


This chart is a virtual "awards show of distinguished and dignified "peanuts." Look who's here.

Of course at the top of the chart is still that main honoree, zero-one.


Cascading below it are the "peanuts" which include the numbers of the " 4 Steps," $12,24,36$, and 48. (Their reflective mates aren't all included, but they are all included in the ( $3-+3+3 \ldots$ sequence) chart.


Over along the left edge we find 6 and 7. Geometrically, this pair expresses the first manifestation of closest packing of circles, the " 6 around $1=7$ " pattern. (Seen as spheres, this is also the "closest packing of spheres on a plane" pattern).


The next, very dignified pair is 12 and 13 . Geometrically, this is the closest packing of spheres, that " 12 around $1=13$ " pattern, which naturally creates a cuboctahedral shape.


And when a second layer (of 42) is closepacked around that 13 , it makes a total cluster of 55 spheres, which might be expressed as " 54 around 1."

Next, we have 18 and 19. These might not be as recognizable as the previous pairs, but a geometer practiced in sphere-assemblies would recognize this as an expression of what I call a "second stage" octahedron.

A "first stage" octahedron has four spheres on a plane, arranged as a square, with an additional sphere nested on top and another nested on the bottom.


A "second stage octahedron has a central grouping of 9 spheres on a plane, arranged in a $3 \times 3$ square. Nested on top of that are 4 more spheres, and that is topped by another single sphere. The same arrangement takes place below the 9 spheres.

This $1+4+9+4+1$ arrangement sums to 19 spheres. But deep in the heart of this arrangement is one central sphere, (the center sphere of the $3 \times 3$ grid). If this is the nucleus, then there are 18 spheres in the assembly which surrounds it. Yes, the " 18 and 19 " pair is very dignified indeed.

You'll note that there is no central sphere in the "first stage" octahedron, nor will there be one in a "third stage" octahedron which has a $4 \times 4$ grid of 16 spheres in its midsection.


The next honoree is the pair " 24,25 ." These are key numbers in the design of a cuboctahedron. All cuboctahedrons, regardless of the size, have 24 edges. To build a cuboctahedron you need 24 same-length sticks. No more, no less.

But a "fourth stage" octahedron does have a middle sphere in the center of its 5 x 5 grid of 25 spheres. Above and below this group of 25 are groups of $16,9,4$, and 1 . This makes a grand total of 84 spheres that are accumulated around 1 central sphere, totaling 85 . You'll notice that $(84,85)$ is also an illustrious member of this $(6+6+6 \ldots$ sequence, $1+6+6+6 \ldots$ sequence) chart!

If you had a cuboctahedron made of rubber and blew it up like a balloon, you would make what is called a "spherical cuboctahedron.

The straight edges are now all curvy to fit around the curvature of the sphere. The lines connecting the vertices are still the shortest-possible lines connecting two points and thus form "great circles" which go all the way around the spheres.

(The equator is a "great circle" which divides the globe into two hemispheres, but any circle around the globe that divides it into 2 exactly equal half spheres is a great circle.)

As a navigational expert, Dee knew all about great circles. He would know that in order to connect every vertex to every other vertex on a spherical cuboctahedron, 25 great circles are required.

So 24 and 25 are numbers which are quite descriptive of the cuboctahedron (which gets its shape from " 12,13 " spheres, and whose midsection center is " 6,7 " spheres).

Dee calls attention to the pairs " 12 , 13 " and " 24,25 " in both of his summarizing charts:

In the "Thus the World Was Created" chart these pairs are in a stack of a "quaternary" of the rectangles, in which two of the rectangles have been left blank.


If Dee he had filled them, he would have been providing too good a clue. But it's pretty obvious now that they should be filled with " 6,7 " and " 18,19 ." The benefit of omitting these pairs is that the " 12,13 " and " 24,25 " both specifically relate to the cuboctahedron.

We just saw that the pattern of " 6 around $1=7$." relates to the chart. But the chart's integration with "closest packing of circles" goes a whole lot deeper than that.


The next layer of circles that fits around these " $\mathbf{6}$ around $\mathbf{1}$ " contains $\mathbf{1 2}$ circles. The next layer contains 18, and the next contains 24. Each consecutive layer is 6 spheres larger than its predecessor. (One might imagine each 6 additional spheres being added to each 6 corners of the hexagon in successive layers.)

In a nutshell, the number of circles in consecutive layers of "closest packing of circles" and the " $6+6+6$...sequence" are the same thing!

There's an amazing correspondence with the cumulative totals of "closest packing of circles," as well.

After the first layer there are 7 spheres. After the second layer there are $1+6+12=19$ spheres. After the third layer there are $1+6+12+18=37$ spheres.

After the fourth layer there are 61 spheres and after the fifth layer there are 91 spheres.

Needless to say, all these results,

$$
7,19,37,61, \text { and } 91
$$

are members of the " $1+6+6+6 \ldots$ sequence"
(the " 1 " referring to the central sphere).

To summarize, there's a distinguished cast of "pairs of characters" in the " $6+6+6+6 \ldots$..sequence, $1+6+6+6$...sequence" chart. They all put on stellar performances, but the "best of show" must go to the pair at the "top of the chart," the zero-one, the source of all this retrocity.

These honorable geometric dignitaries are evidence that the energy of oppositeness pervades "number" as well as "shape," a powerful demonstration that arithmetic and geometry are two sides of the same coin. Or as Dee cryptically infers: there is a MET in geoMETry and in arithMETic, the 2 main branches of MathEmaTics.

In his Letter to Maximillian, Dee writes about poeple who study sphere assemblies:
"those who have diligently presented their findings regarding
PLENUM, occupied by matter, and VOID, empty of matter...
They have seen that the surfaces of Elements, which are in close proximity are coordinated, connected, and Joined together by a Law (decreed by God Almighty)
and Bond (practically unable to be loosened) of Nature."
(Dee, Letter to Maximilian, p. 6 verso)

Dee was no doubt enthralled to discover all this harmony of number and geometry. He writes that his mind was pregnant with the Monas Hieroglyphica for 7 years ( 2520 days) but it's clear he had investigated this subject as early as the 1540's when he discussed it with his buddy Gerard Mercator in the Louvain.

He was so excited about it, he synthesized as not only into the form of a book, but also into the form of an architectural structure.

Thus, hidden deep within the main proportions of his tower are not only the " 4 steps," $12,24,36$, and 48 , but also natural geometric concepts like the "closest packing of circles," the "closest packing of spheres," and the "cuboctahedron"!

Standing in Touro Park today, you'll see a weather-worn, half-destroyed remnant, but with a little imagination you can see a giant cluster of 55 spheres in the shape of a cuboctahedron! The " 54 around 1 " shape is approprite for this metaphor because the Tower, most likely, was 54 feet tall (including the finial atop the dome).


## Three sequences meet 55.

Three wonderful series intersect at this 55 nexus: the vertical "11Wave,"

the offset, but horizontal " $1+9+9+9 \ldots$ sequence"
and the Hitching Sequence.

Here's how the first two intersect:


We've also seen that the Hitching sequence is centered on 55 .

This suggests that the Hitching sequence is deeply entrenched with retrocity. To see if this is so, let's multiply the members of the Hitching Sequence by 99 .

Suddenly more transpalindrimes appear!
For example,

$$
" 13 \times 99=1287 "
$$

and $79 \times 99=7821$."
The reflective mate of 1287 is 7821 . Who would ever suspect that 13 and 79 were somehow related to each other?

Or 17 and 39?
Or 31 and 97?
or 71 and 93 ?


Ignoring transpalindromicitiy for a moment, let's look at the "averages" of these results. Various pairs sum to 10890 , which, when divided by 2 , makes 5445.


There is one pair of result which is not transpalindromic: 3663 and 7227 . However, these two do sum to 10890 , and average to 5445 , like all the rest .

All thus "summing to 10890 " and "averaging to 5445 " is not really surprising, because basically we've just taken the whole chart and multiplied everything by 99 . But the important thing is that 10890 It's also the lead member of the fourth great wave that pulses through the realm of number. First there's the 9 Wave (which works with the 11 Wave), then the 99 Wave, then the 1089 Wave, then the 10890 Wave.

## Remember, all we started out with here is a handful of prime numbers that were reflective, and suddenly theres a connection to the 10890 Wave of Consummata.

You might not wake up in the morning thinking about the 5-ditit range of number, but you probably use them every day-in zip codes.

I was hoping that some obscure Amercan town would gain no-
 toriety by having the zip-code 10890 . However, only about 43,00 of the 99,999 possible 5 -digit zip codes are currently in use, and 10890 is not one of them. But a few towns do have zip codes that are in the 10890 Wave. Congratulations to the folks in Matoon, Wisconson, who on the "nave."

Back to our story. The four new pairs of transpalindromic mates suggesting third way of connecting the members of the Hitching sequence.


Superimposing all 3 sets of interconnecting lines makes a graphic representation of a cross.


Let's superimpose this "graphic cross" onto the intersecting vertical "11 Wave" and the horizontal " $1+9+9+9$... sequence," which also intersect at 55.



The "offset cross" is clearly evident when compared to the "equilateral cross" made by the 9 Wave and the 11 Wave.

## Sound familiar?

The Monas symbol has an "offset cross" as well! The geometric proportions of this "offset cross" and Dee's "Cross of the Elements" are not precisely the same. (The proportions of the Cross of the Elements is involved in other stories as well.) But, metaphorically speaking, an "offset cross" is any cross that is not equilateral. For example, in Theorem 5, Dee infers that his "offset cross" or an "equilateral cross" or an X-shape can all represent the Roman numeral for 10. (In Theorem 21, he even inverts the Cross. In the "36 Boxes" diagram of Theorem 22, he hints at 2 Crosses,"Crux' and "Crux", later referred to as "Octonarius our Crosses").

> I'm suggesting that Dee saw these very same patterns in the diamond-shaped chart and is speaking about them metaphorically in the text and graphics of the Monas Hieroglyphica.

The nave of the "offset cross" falls on a whole number, 55. The nave of the equilateral cross does not fall on a whole number (it falls on 49.5).

This is what Dee is metaphorically referring to in Theorems 5 and 20 regarding the Cross being Ternary ( 2 lines and a point, where the point is considered to be present) and Quaternary (4 lines, where the point is not considered to be present).
Dee is speaking to readers who have seen the 2 distinctly different naves in the perfect reflective symmetry of the diamond-shaped chart of the 1-digit and 2-digit numbers.
"Offset cross/equilateral cross," "point/no point," "9 parts/10 points," are all interwoven in Dee's Theorems, just as they are on this diamond-shaped chart-if seen properly. The Monas symbol is so much more than a squatting, one-eyed, horned homunculus!

## MORE AmAZING Syndex Numeronomy: the "Special 4- Palindromes"

If you've followed this discussion so far, you are developing into a fine "numeronomist." This chapter is "advanced numeronomy." It involves another sequence of number that Marshall found, which is important in that provides another example of the synchrony between Holotomes and the Cycloflex.

Dee does not appear to make reference to this sequence which joins Metamorphosis and Consummata. He probably knew of it, but to include it in his short, cryptic treatise would only tend to confuse the reader.

It involves the $(1+9+9+9 \ldots$ sequence $)$, most of which is in a horizontal line.
"Special 4-digit Palindromes"

sequence
$10 \times 99=0990$
$19 \times 99=1881$
$28 \times 99=2772$
$37 \times 99=3663$
$46 \times 99=4554$
$55 \times 99=5445$
$73 \times 99=7227$
$91 \times 99=9009$
$1+9+9+9 \ldots$
sequence
$1+9+9+9 \ldots$
sequence
$10 \times 99=0990$
$10 \times 99=0990$
$19 \times 99=1881$
$19 \times 99=1881$
$28 \times 99=2772$
$28 \times 99=2772$
$37 \times 99=3663$
$37 \times 99=3663$
$46 \times 99=4554$
$46 \times 99=4554$
$55 \times 99=5445$
$55 \times 99=5445$
$64 \times 99=6336$
$64 \times 99=6336$
$73 \times 99=7227$
$73 \times 99=7227$
$82 \times 99=8118$
$82 \times 99=8118$
$91 \times 99=9009$
$91 \times 99=9009$

When these numbers are each multiplied by 99, Marshall's "sequence of "Special 4-digit Palindromes" is created.

Each of these palindromes
might be paired together with its
"inside out" friend, like this:



18818118
$\begin{array}{ll}2772 & 7227\end{array}$
36636336
45545445

The "Special 4-digit Palindromes" are connected to with Consummata in several ways.



Another way to see interconnection between the "Special 4-digit Palindromes" and Consummata is by adding some of the members of the 1089 Wave to the some of the members of the 99 Wave.

So it's pretty clear that this sequence is connected to Consummata, but how is it connected to Metamorphosis?

Strangely enough, it's connected by way of Metamorphosis number $\mathbf{3 6 0}$.

Here's a chart of the multiples of 360 added to their reflective mates. It's amazing how many of the results are members of the group "Special 4-digit Palindromes."

 between this sequence and Metamorphosis numbers in the " 108 wheel."

Marshall created a "108 Wheel," which spiraled out to number 6480. butthis one only shows the first 4 spirals, ending to 432.

As the numbers spiral clockwise, 108 and its multiples can be found at in the top radian. I have highlighted the 11 numbers in the first spiral that form the "9 Wave" and also the 12th number, $\mathbf{1 0 8}$ (as 108 is $9 \times 12$ ).


The ancient Hindu timekeeping "Yuga numbers" 432, 864, 1296, and 1728 are all multiples of 108 , and are found in that top 108 radian (if continued).

All the Metamorphosis numbers (except 12 and 24, which are too small) can be found in either in the radian that begins with 36 or in the radian that begins with 72. Even the Exemplar Number, 12252240, is in that radian which begins with 72. (I've also shown 2 very large Metamorphosis numbers to suggest that this pattern continues indefinitely.)

Thus, in the 108 wheel, the Metamorphosis numbers and the "multiples of 108 " are in a "triangular array."

The other amazing thing that Marshall found while contemplating the 108 Wheel is that all of the special "Special 4-digit Palindromes" are found in only 4 of the radians, those starting with $18,45,72$, and 99 . (Curiously, subtracting 45 from 72 results in 72 's reflective mate 27 ). These 4 radians are 27 radians apart from each other (as $27 \times 4=108$ ).

Thus these radians are in "quadratic array," in the sense that they form a square.


Note that the triangular array and the square array share a corner, the radian that starts with 72.


The 108 Wheel is a powerful tool to help investigate the synchrony of Consummata, Metamorphosis and these "Special 4-digit Palindromes." And of course 108 is intimately interrelated with Dees Magistral number, 252, as $108+252=360$.

Further out this 72 radian (in the 25th spiral of this chart) that "Special 4-digit Palindrome" 2772 can be found. In this same radian (in the 256th spiral of the chart) the Metamorphosis number, 27720 can be found. ( We'll hear more about these characters in a moment)

Congratulations. If you have followed this wild story of the synchrony, you're now an advanced numeronomist, and your job is to explore all this further.

# Connecting Consummata to the Pretzel TO METAMORPHOSIS TO THE MONAS SYmbOL (SIMPLY WITH 36 AND ITS BUDDY 360) 

A method I call "x-ray vision" multiplication opens a door to seeing the connection between the 1089 Wave of Consummata, the numbers 108 and 252 of the Pretzel, Metamorphosis number 360 , the " 9 parts and 10 points of the Monas symbol, as well as the overall grid pattern of the Monas symbol. Are you ready?

Here is the 1089 Wave being created by "the 11 Wave times 99", shown in "traditional" long multiplication.


Look at the numbers in the "middle section" of this long multiplication (99, 198, 297, $396,495,594,693,792,891)$. They are all members of the 99 Wave.


My "x ray vision" multiplication method is just a little different. When multiplying to make the "middle section," I don't do any "carrying over." Now, all the numbers in the "middle section" are members of the 9 Wave $(9,18,27,36,45,54,63,72,81)$.

Allow me to do one more "creative" multiplication. We've used the "alliterative" members of the 11 Wave like twenty-two, thirty-three, forty-four,... eighty-eight, and ninety-nine. Let's also use "tentyten." (There's a method to my madness.)


The various sums of those "middle sections"

Finally, let's add the results in the various "middle sections."
Ta-da!!
(Sound of fanfare)
The resulting numbers are the elegant
"multiples of 36,"
culminating in that great Metamorphosis number $\mathbf{3 6 0}$.

This is simply " 36 times $1,2,3,4,5,6,7,8,9$, and 10 ," all the members of the Decad. As the Decad is symmetrical when " 10 is seen as a return to 1 ," let's extrapolate and consider " 360 to be a return to 36 ." In other words, let's triangulate these "multiples of 36 ," like this:

Notice anything special? At one of the lower corners is
 252, Dee's Magistral number. The other corner is 144 , notable for being 12 squared. These are two key members of the " 12 and 21" Pretzel!

Also, each side of the triangle has 108 parts, another key number of the " 12 and 21 " Pretzel.



And since the Monas symbol represents the Symmetry of the Decad, we might apply the "multiples of 36 to the Monas symbol. Suddenly, the intersection point of the Cross of the Elements is 144 , and the point in the center of the Sun Circle is 252 . What a spectacular place for Dee's Magistral number!


But remember, Dee shows two versions of the Monas symbol, the upright version and its "opposite," the inverted Monas symbol.

The opposite of 36 is, of course, 63. This suggests we might see the inverted Monas symbol as the "multiples of 63 ," up to 630 .

Now, the intersection point of the Cross of the Elements is 441, (where previously it was its opposite, 144). And the centerpoint of the Sun Circle is 252 , exactly the same as it was before! That's pretty amazing.


Here's what the "multiples of 63 , up to 630 " look like "triangulated." The lower corners are 252 and 441

Each of the 3 sides has 189 parts, another key number in the "12 and 21" Pretzel.


In short, Dee's 2 versions of the Monas symbol and Marshall's Pretzel are both expressions of the same thing!

The star of each of these graphical treatments is Dee's Magistral number, 252. The transpalindromes, 144 and 441, play a strong supporting role, as do 108 and 189.


In discussing the Pretzel earlier, I referenced Dee' quote in Theorem 17. After arriving at 252 by adding $20+200+10+21+1$, he writes:

There are two other logical ways
that we can draw forth this Number from our premises.
For the sake of brevity, we recommend these reasons be rooted out by Beginning Cabalists.
The various artificial productions of this Magistral Number are also worthy of the Consideration of Philosophers.

The "artificial [skillful or artful] productions" seem to be the two versions of his very own Monas symbol.

Let's take this a step further:
The Monas symbol is the" 12 and 21 " Pretzel, and the Monas symbol is the basic design plan of the John Dee Tower, then is the " 12 and 21 "Pretzel is the John Dee Tower,
the star of which is $\mathbf{2 5 2}$.
All I can say is WOW MOM!!!

## 252



Boiling it down even further, these 3 things are expressions of $\mathbf{2 5 2}$, the product of the first palindromable pair of numbers, $\mathbf{1 2}$, the fabulous "docena" and its reflective mate, 21. The "Magistral" 252 , is also the transpalindromic mate of 2520.

As we just applied the " 10 is a return to 1 " idea to " 360 is a return to 36 ," and " 630 is a return to 63 ," and found interesting results, an obvious next step is to investigate " 2520 is a return to 252 ."


You might recognize 2772 as "one tenth" of Metamorphosis number 27720 (as $2520 \times 11=27720$ ). And it's also one of the "Special 4-digit Palindromes" (as the "1+9+9+9... sequence" number 28, times 99 , equals 2772).

We'll see in a moment just how special 2772 is. It might even be condidered the 252 of the 4-digit numbers range.

I'll give you a taste of how well-connected it is. Remember the numbers 396 and 693 which appear at the bottom of the Extended Pretzel, and are in the $4: 7$ ratio? $(252+144=396$ and also $252+441=693$ ). Multiply 7 times 396 and the result is 2772 . Multiply 4 times 693 and the result is also 2772 .

You just know 2772 is got to be special, as its the sum of 2 very special numbers, 252 and 2520,

A quick review: using "x-ray multiplication to analyze the 11Wave x 99 makingthe1089 Wave, we found "the multilpes of 36, up to 360."
"Triangulating" these 10 numbers highlighted 144 and Dee's Magistral number 252.After also triangulating the "multiples of $\mathbf{6 3}$, up to $\mathbf{6 3 0}$," we had found all the members of the " 12 and $21 "$ Pretzel (252, 144, 441, 108, and 189).

The lowest of these special numbers is $\mathbf{3 6}$, an essential ingredient this concoction of numbers. The number $\mathbf{3 6}$ has a lots of important freinds.

36 is the sum of the first 2 Metamorphosis numbers $(12+24=36)$.
36 is also half of Metamorphosis number 72.
36 is also the third step of the " 4 Steps"," $12,24,36,48$.
36 can sing those 3 great harmonies, $1 / 2,2 / 3$, and $3 / 4$, when accompanied by its friends 18,24 , and 27.
36 also sings these harmonies with his "larger" set of friends 72,54 and 48.
(Of these numbers, " $18,27,54$, and 72 " are in the 9 Wave.
The others, 24 and 48 , are buddies with 36 in the " 4 Steps.")
36, and its reflective mate, 63 are a dynamic duo.
Together they sum to 99 .
Together they form that awesome $4: 7$ ratio that pervades the " 12 and 21 " Pretzel and the " 4 Steps."

Dee wrote $\mathbf{3 6 \times 7 = 2 5 2}$ in his own handwriting in the margin of his copy of Voarchadumia, next to where Pantheus says the "number of days" is 252 .
This multiplication, $36 \times 7=252$, is the essence of $7 \times 360=2520$
(the lowest number divisible by all the single digits).
" 360 relates to 36 " the way " 2520 relates to 252 ." In the " 10 is a return to 1 " way.
Dee cryptically made a big deal out of 360 by making the "ballooned 360" a huge clue in cracking the code of the " 2 circle geometry" of the "Thus the World Was Made" chart.

Dee emphasized 36 in the Monas in subtle, but poignant ways.
In Theorem 22, the largest illustration in the whole book is what I call the " 36 Boxes" chart. This is a clever way for one geometer to shout " 36 " to other geometerswithout anyone else hearing.

An even more important geometric expression of $\mathbf{3 6}$ is the 4 X 9 grid upon which the Monas symbol is based.
Ultimately, 36 (and thus 360) gets muchof its "Metamorphosis character" by bieng related to12, the powerful docena,
which Dee tells us in the the Artificial Quaternary, gets it's energy from, 1, 2,3 and the "essence of 4." It gets Consummata character" by being related to that "pause in the rhythm of ocaves," null number 9 .

So despite the fact that I have introduced the "multiples of 36 , up to 360 " using creative multiplication technique, once found it opens the door to a wonderful view of the synchrony between Consummata and Metamorphosis as seen via the Pretzel and the Symmetry of the Decad.

## The Big Picture OF HOW THE "12 AND 21 PRETZEL" relates to Consummata

To see how the "multiples of 36 , up to 360 " and it's cohort, the "multiples of 63 , up to 630 " fit into the big picture of Consummmata, let's apply this " 10 is a return to 1 " idea to all the members of the 9 Wave, 99 Wave, 1089 Wave, and beyond. First, here's an overview of these Waves.


To briefly recapitulate, null number 9 and null number 90 embrace an octave of transpalindromic members of the 9 Wave. Null number 99 (made when the 11 Wave meets the 9 Wave) and null number 990 embrace an octave of transpalindromes in the 99 Wave. In the 1089 Wave, a nave suddenly appears, separating the octave of transpalindromic mates. This same pattern continues in the 10890 Wave, the 108900 Wave, and beyond.

This next chart shows "first 10 multiples" of all the numbers on the Consummata chart. The 10 multiples are associated with the " 10 points of the Monas symbol" to emphasize the numbers associated with the Cross of the Elements centerpoint and the Sun Circle centerpoint.

Notice that in each octave, the first " 4 shapes" are upright Monas symbols and the second "4 shapes" are inverted Monas symbols, graphically expressing the "oppositeness" character of the transpalindromes. I've flopped the null numbers 9 and 90 (as well as 99 and 990) onto their sides to suggest "nullness." But as they are still transpalindromic mates, I've oriented them in opposing directions (their "Moon semi-circles" seem to become parentheses).

See if you can locate the "12 and 21 Pretzel" in this "big picture."

## Consummata meets Symmetry of the Decad



10890 Wave

...)


There are hundreds of numbers in this flip-flopping chart. Let's start in the upper lefthand corner with the sideways Monas symbol of the "multiples of $\mathbf{9}$, up to $\mathbf{9 0}$." Note that the Cross centerpoint is 36 and the Sun Circle centerpoint is 63 .

Three Monas symbols further to the right is the "multiples of 36, up to 360" upright Monas symbol, whose Cross centerpoint is 144 and whose Sun Circle is 252.

Three more Monas symbols to the right is the "multiples of 63 , up to 630 " inverted Monas symbol whose Cross centerpoint is 252 and 2 whose Sun Circle centerpoint is 441 .

Three more Monas symbols to the right is the "multiples of $\mathbf{9 0}$, up to $\mathbf{9 0 0}$," sideways Monas symbol. It's Cross centerpoint is 360 and it's Sun Circle centerpoint is 630.

In other words, the first, fourth, seventh and tenth Monas symbols are all related to the Pretzel. To express this graphically requires a full-page illustration, but it's worth it, in order to "snap a picture" of all this synchronicity.

Here are two other ways of expressing what's going on here: the basic "12 and 21 Pretzel" and the " 10 is a return to 1 " type triangulations."


Seen as a triangle, and $\mathbf{1 8 9}$ are the numbers of parts on each of the sides.


Indeed, the whole Consummata chart could be redrawn using "trianglulations" instead of Monas symbols, (but it just gets graphically cumbersome).

Its hard to envision a triangle and "the spine of the Monas symbol" as the same thing, so here I've added a coulpe of intermediary steps.


When Dee signed his name with a triangle, it meant more than just the "fourth Letter of the Greek alphabet"; it also meant the Monas symbol.

But wait, there's more!
The numbers 108 and 189 appear in another place.

In the "multiples of 27, up to 270 " Monas symbol, the Cross centerpoint is 108 , and the Sun Circle centerpoint is 189.


Remember, Consummata involves octaves (the number 8), and the "multiples of 9 " and be "multiples of 11 ." It doesn't really seem to involve 12.

Yet here, in the heart of Consummata are numbers relating to 12 . We find " 12 squared" (144), the product of " 12 and it's mate 21 " (252), the "square of its mate" (441) as well as the differences between these numbers (108 and 189).

We've also seen that the number 12 is at the heart of Metamorphosis, being the first Metamorphosis number.


Thus, the number 12 seems to be at the heart of the synchrony be- tween Consummata and Metamorphosis.

Also, remember that the "delightfully dividable docena" is the most highly composite number for its size (divisible by $2,3,4$, and 6 ). But it's also famous for being the first transpalindromable number ( $\mathbf{1 2}$ and 21).

It's even more spectacular because the squares of 12 and its mate 21 are also transpalindromes. The only other pair of numbers that can make this claim are for 13 and 31 . They multiply to 403 and their squares are 169 and 961 . But, you don't find 403, 169, and 961 among these hundreds of numbers on the Consummata chart. Nor will you find their differences (234 and 558) or any of the numbers on the 13 and 31 Extended Pretzel. When it comes to interacting with other numbers, thirteen ain't no docena.


The numbers generated by " 12 and 21, " ( 144, 252 , and 441) are not only present but they are at key fulcrum points in the whole arrangement of the 9 Wave.

Not only are the 36 and 63 in the corners of the 9 Wave triangle, but 144,252 , and 441 are in the corners of the "multiples of 36 , up to 360 , and the "multiples of 63 , up to 630 " triangles.

But wait, there's even more!!
In the last chapter we saw that the various pairs of the "multiples of 252 , up to 2520 " sum to that palindromic number 2772 . The most noteworthy summation is " $252+2520=$ 2772."

Besides being one of the "Special 4-digit Palindromes" (as $28 \times 99=2772$ ), 2772 is also one tenth of Metamorphosis number 2520 .

$$
2772
$$



In the chart of "Consummata meets of the Symmetry of the Decad," 2772 pops up in two places, both in the 99 Wave.

And these two places are ultra-special because they are the same positions where 252 appeared in the 9 Wave!


This calls for another larger diagram to get a graphic sense of what's going on here. Here, I've enlarged the first Monas symbol on the left side of the 99 Wave ("the multiples of 99 , up to 900 ").

It's Cross centerpoint is 396 , and it's Sun Circle centerpoint is 693. Recall that these are part of the Extended Pretzel (as $144+252=$ 396 , and $252+441=693$ ).

I've also enlarged the last Monas symbol on the right, the "multiples of 990, up to 9900 ." Its "fulcrum points" are 3960 and 6930.



In the "same positions" where 144 and 441 appeared in the 9 Wave, 1584 and 4851appear in the 99Wave. Amazingly (or perhaps it should have been expected) these two are transpalindromes.

The numbers 1584,2772 , and 4851 occupy the "same positions" as 144,252 , and 441 because they are " 11 times" these numbers. Their makeup is clearer when you see the "middle section" of the long multiplication.

| (28 is a member |
| :---: |
| of the $1+9+9+9 \ldots .$. sequence) |

$x \frac{28}{99}$
$\frac{252}{252}$
2772
(2772 is a
"Special 4-digit Palindrome")


| 144 | 252 | 441 |
| ---: | ---: | ---: |
| $\times \frac{11}{144}$ | $\frac{11}{252}$ | $\frac{11}{441}$ |
| $\frac{144}{1584}$ | $\frac{252}{2772}$ | $\frac{441}{4851}$ |

It's also useful to see"middle sections" the long multiplication of the creation of "Special 4-digit Palindrome 2772," as well as the creation of Metamorphosis number 27720 from 2520.
1584:4851 = 16:49 = (4x4):(7x7) important 4:7 ratio in another way. The number 1584 is $16 \times 99$, and the number 4851 is $49 \times 99$. From this we can see that $1584: 4851$ is equivalent to $16: 49$, which you will recognize as " 4 squared: 7 squared."

This isn't exactly $4: 7$, but they have plenty of $4: 7$ gusto in their relationship with 2772 :

$$
1584: 2772=4: 7
$$

$$
2772: 4851=4: 7
$$



Dee makes no overt references to 2772 in the Monas Hieroglyphica, only subtle ones. We've seen that in the "Thus of the World Was Created" chart, the "Supercelestial" arc represents 72. To anyone aware of the reflectivity of number, when they saw 72 they would also think of 27 .

## 27 A 72

Remember that the Metamorphosis numbers are the relative proportions of the areas of the 3 small circle-segments. The largest segment must "balloon" to the area of "half-circle segment" to make Metamorphosis number 360.

As $12+24+72=108$, the 360 half-circle segment "needs" 252 . This is a geometric expression of an equation found in the "Extended 12 and 21 Pretzel."


Engraved on the foundation of the architecture of the Title Page is a quote from the Bible which "just happens" to be a line from Genesis 27 . Again, when anyone savvy to the reflectivity of numbers saw 27 they would also think of 72 .

The association between 27 and 72 is more easily recognized then between most other pairs of transpalindromes (say for example 46 and 64 , or 38 and 83) because 27 and 72 are important members of the 9 Wave. (And they are next-door neighbors of that other very famous couple 36 and 63).


To summarize, there are so many numbers and interweavings in this story, it can get confusing. The "Consummata meets Symmetry of the Decad" chart is a graphic way to see that the "12 and 21 Pretzels" numbers are at the very core of Consummata.

Dee expresses the mathematical Symmetry of the Decad $(1,4,7,10)$ in a graphic geometrical way by creating the Monas symbol. It's a tool, a measuring stick, to help paint a clearer picture of these marvelous numbers and their interconnections.

Let's bring these mathematical meanderings closer to home. As the John Dee Tower was built on the plan of the Monas symbol, it has the " $1,4,7,10$ " Symmetry of the Decad implicit in its design.

The"1, 4, 7,10" proportioning of the Symmetry of the Decad


As the "12 and 21 Pretzel is intertwined with to the Symmetry of the Decad, the Tower has all these numerical relationships implied in its design as well. Prominent among these is Dee's Magistral number, 252.


Furthermore, the Tower has all the numbers of the
"Extended Pretzel"latent in its Monas symbol design.


Further furthermore, it has the numbers related to "Special 4-digit Palindrome" 2772 implicit in its design.


The "equals sign" in the preceding diagrams is slightly metaphorical. It means that the "proportions" of all of these items are the same. But none of these Monas symbols depicts the actual measurements of the Tower in feet.

The stone and mortar part of the tower was 48 feet tall, with a 6-foot finial or flagpole on top, making it 54 feet tall. These numbers don't appear on any of these key diagrams.

However, the 54 -foot height of the Tower is actually quite related to the "multiples of 36, up to 360 " Monas symbol. Here's how.

The number 36 can be seen as " $2 \times 18$," " $3 \times 12$," or " $4 \times 9$," but perhaps most special set of divisors is " $6 \times 6$ " (the "roots "of 36). The Tower design is based on this " $1 / 6$ of 36 ," in other words, "the multiples of 6 , up to 60 ."


Now look at the the Cross centerpoint and the Sun Circle centerpoint! They are Metamorphosis number 24 and its transpalindromic mate 42 , one of the " 4 Steps" (each of steps in the ratio of $4: 7$ ).

But this makes it seem as if the Tower was 60 feet tall, not 54 feet tall! What's up with that?

The reason for this apparent discrepancy is that we "we measure in parts, not points." In other words, when you look at a ruler, the " 1 inch mark" isn't on the left edge of the ruler, it is 1 inch to the right of the edge. It's counting the "part." (If it were counting "points," it would be a 2.)



If we drew a Monas symbol with 10 points spaced 1 inch apart from each other, it wouldn't be 10 inches tall, it would only be 9 inches tall.

Remember, this discrepancy between 9 parts and 10 points is what is part of what Dee sings about in the "round" of the "flowing ribbons": Mercury (9), when made perfect by a sharp, stable point (1), becomes the King (10) of all the planets.

Well, the same sort of thing applies to the "multiples of 6 , up to 60 ."


The uppermost point (60) minus the lowermost point (6) makes 54. This triangular representation shows how 60 is a return to 6 ," in the way " 10 is a return to 1 ."


There are 18 parts on each of the 3 sides, totaling 54, the height of the Tower in feet.



Let's take this a step further. A standard unit of measure in Dee's time and in ours is the foot, 12 inches.

I've devised a new unit of measure, which is only 2 inches long (or $1 / 6$ of a regular ruler).

I call it a "twinch."

Suppose we measured the Tower in "twinches," instead of feet. (in other words, all the various heights would be multiplied by 6)

The height of the Tower would be 324 twinches. ( 54 feet times " 6 twinches per foot" equals 324 twinches.)

I'm not suggesting that Dee made up this new strange unit of measurement called the "twinch." I'm sure he would have liked to have built a tower 324 feet tall. But, as that would not have been feasible, he divided all of his celebrated numbers by 6 making a 54 foot tall tower.


Remember this "multiples of 36 , up to 360 " Monas symbol, (along with its inverse, "the multiples of 63 , up to 630 ") is at the very heart of Consummata. They encapsulate the Pretzel, which is derived from the first pair of transpalindromable numbers, " 12 and 21 ." The number 12 is also at the very heart of Metamorphosis. Thus, the John Dee Tower, when measured in


The John Dee Tower as Monas symbol
"twinches," expresses the synchrony between Consummata and Metamorphosis.

Dee has sculpted a mathematical, geometrical building as rare as the Hope diamond. As it conforms to "nature's laws" of number, which Dee rediscovered, we might consider that the John Dee Tower is one of the most natural, organic buildings ever constructed on Earth. That's quite a claim for a weather-beaten, half-demolished structure that most people consider to be a Colonial windmill, but what we are actually seeing is a "diamond in the rough."

